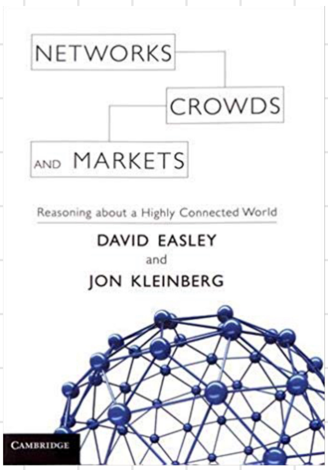
Lecture 9

Network Science

Positive and Negative Relationships

Today's Topics

- Structural Balance
- the Balance theorem
- Applications of Structural Balance
- A weaker form of S. B.
- Generalizing the definition of S. B.



Chapter 5

"Positive and Negative Relationships"

Positive and Negative Relationships

friends and enemies
conflicts, disagreement
controversies, distrusts, ...

Structural Balance Theory:
framework to understand
the tensions between
opposing forces inside
a complex system

local effects can have
a global consequence at
a network level

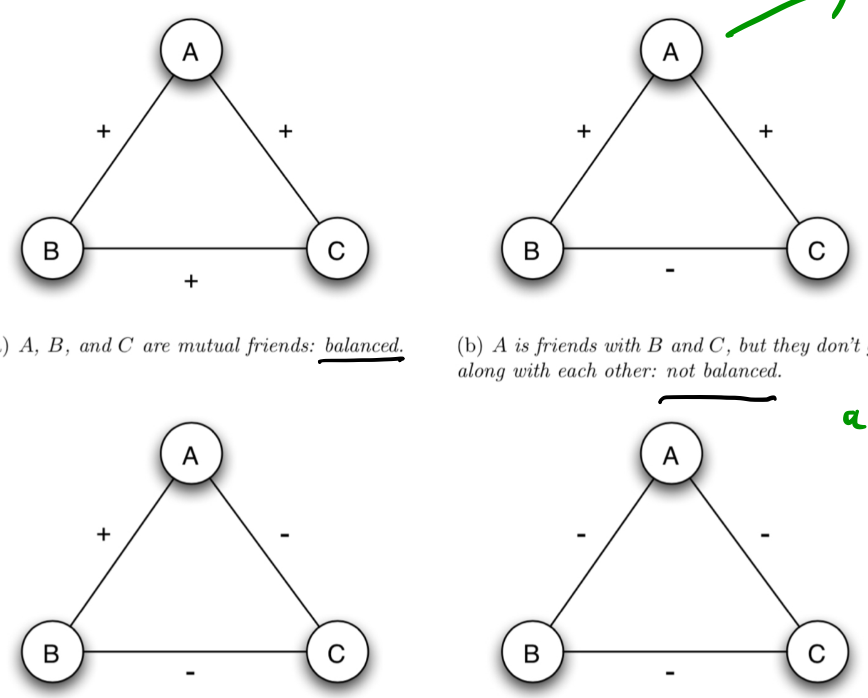
Structural Balance

setting : complete graphs
(cliques)

links : { + , - }

original sources :
- Heider (1940s)
- Harary & Cartwright (1950s)

crucial idea : reasoning on social-psychological behaviours between nodes



(a) A, B, and C are mutual friends: balanced.

(b) A is friends with B and C, but they don't get along with each other: not balanced.

(c) A and B are friends with C as a mutual enemy: balanced.

(d) A, B, and C are mutual enemies: not balanced.

Figure 5.1: Structural balance: Each labeled triangle must have 1 or 3 positive edges.

stressed : will make a choice

$\Downarrow$

will search balance

"the enemy of my enemy is a friend"

$\Downarrow$

signs may change after alliances

Structural Balance Property

P1 : a labeled complete graph is balanced if every triangles is balances

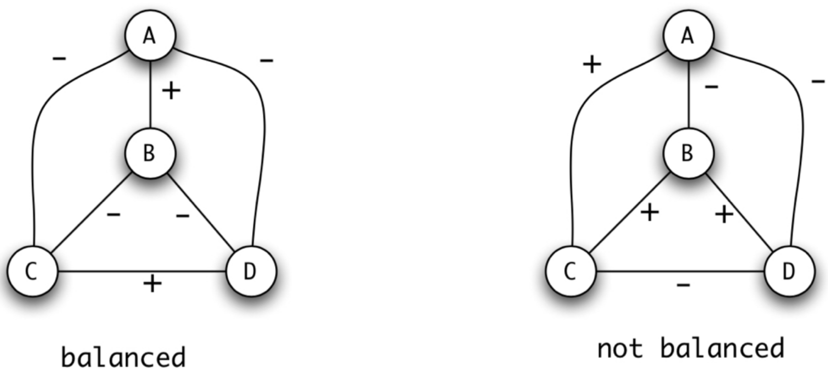


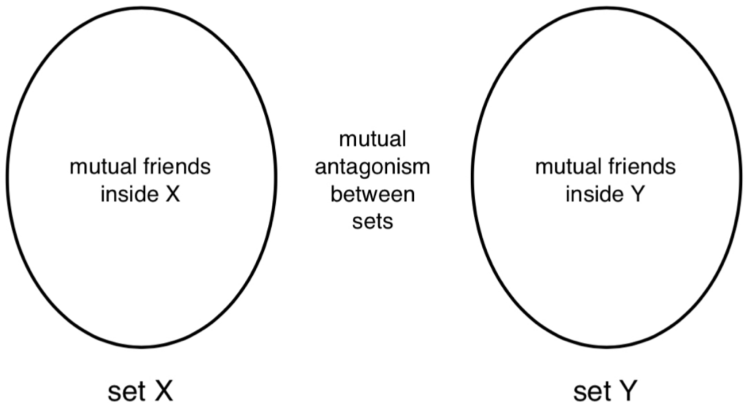
Figure 5.2: The labeled four-node complete graph on the left is balanced; the one on the right is not.

P1 represents a limit of a social system that has eliminated all + unbalanced triangles

unrealistic but useful as a first step

The Balance theorem

if we can divide the graph in two groups where nodes are mutual friends inside each group and enemies are between nodes in the two groups, then the graph is balanced



two opposite factions

Figure 5.3: If a complete graph can be divided into two sets of mutual friends, with complete mutual antagonism between the two sets, then it is balanced. Furthermore, this is the only way for a complete graph to be balanced.

this is the only way for a graph to be balanced.

Proof by Harary (1954)

Proof

Hp.

- a complete labeled graph
- balanced

Ts.

$\Rightarrow$ everyone is friend

or

I will find two groups X and Y

A : node

X : as the set of friends of A

Y : as the set of enemies of A

(i) every nodes in X are friends

(ii) every nodes in Y are friends

(iii) $\forall x \in X$, $\forall y \in Y$

x and y are enemies.

proof (cont'd)

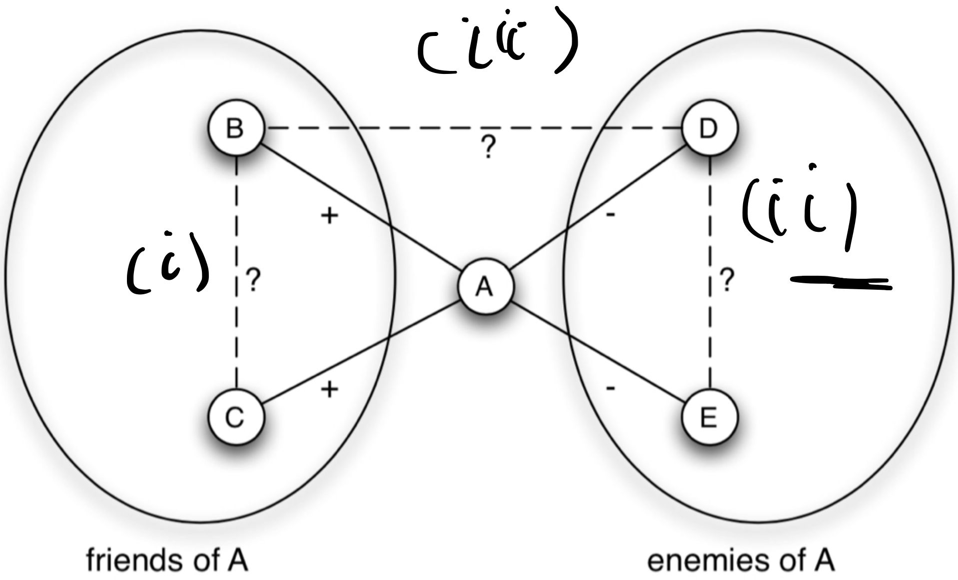


Figure 5.4: A schematic illustration of our analysis of balanced networks. (There may be other nodes not illustrated here.)

$(A, B): +$
$(A, C): +$ & P1 $\Rightarrow (B, C): +$

(i) true

$(A, D): -$
$(A, E): -$ & P1 $\Rightarrow (D, E): +$

(ii) true

$(A, B): +$
$(A, D): -$ & P1 $\Rightarrow (B, D): -$

(iii) true

$\square$

Applications

* "approximately balanced"
* generic graphs aren't complete

But

we want to use this basic notion on complete graph to study dynamical processes.

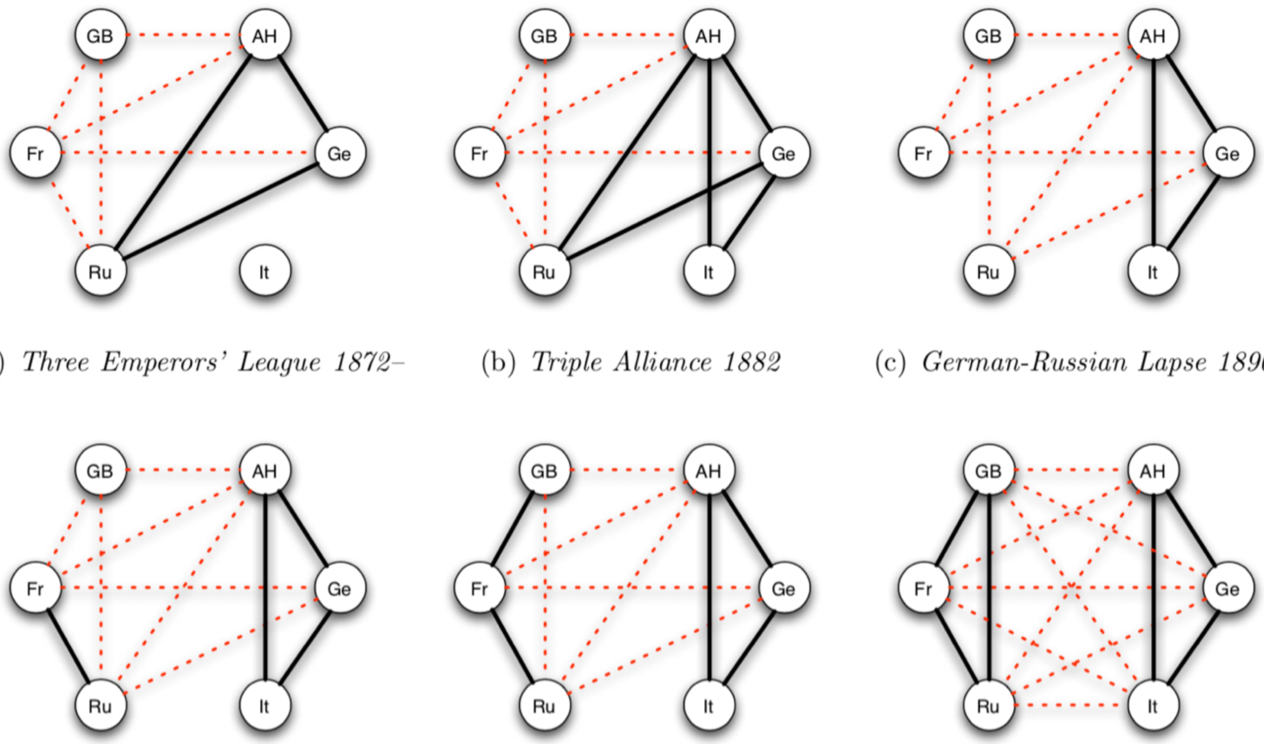
How a complete graph might evolve in the search of balance?

ANALOGY:

physical systems reconfigure themselves to minimize their energy

Some examples follow

International Relations



(a) *Three Emperors' League 1872–81* (b) *Triple Alliance 1882* (c) *German-Russian Lapse 1890*

(d) *French-Russian Alliance 1891–94* (e) *Entente Cordiale 1904* (f) *British Russian Alliance 1907*

Figure 5.5: The evolution of alliances in Europe, 1872-1907 (the nations GB, Fr, Ru, It, Ge, and AH are Great Britain, France, Russia, Italy, Germany, and Austria-Hungary respectively). Solid dark edges indicate friendship while dotted red edges indicate enmity. Note how the network slides into a balanced labeling — and into World War I. This figure and example are from Antal, Krapivsky, and Redner [20].

the search for balance

⇒ a slide to a hard

to resolve opposition

between two sides

Trust, Distrust and Online Ratings

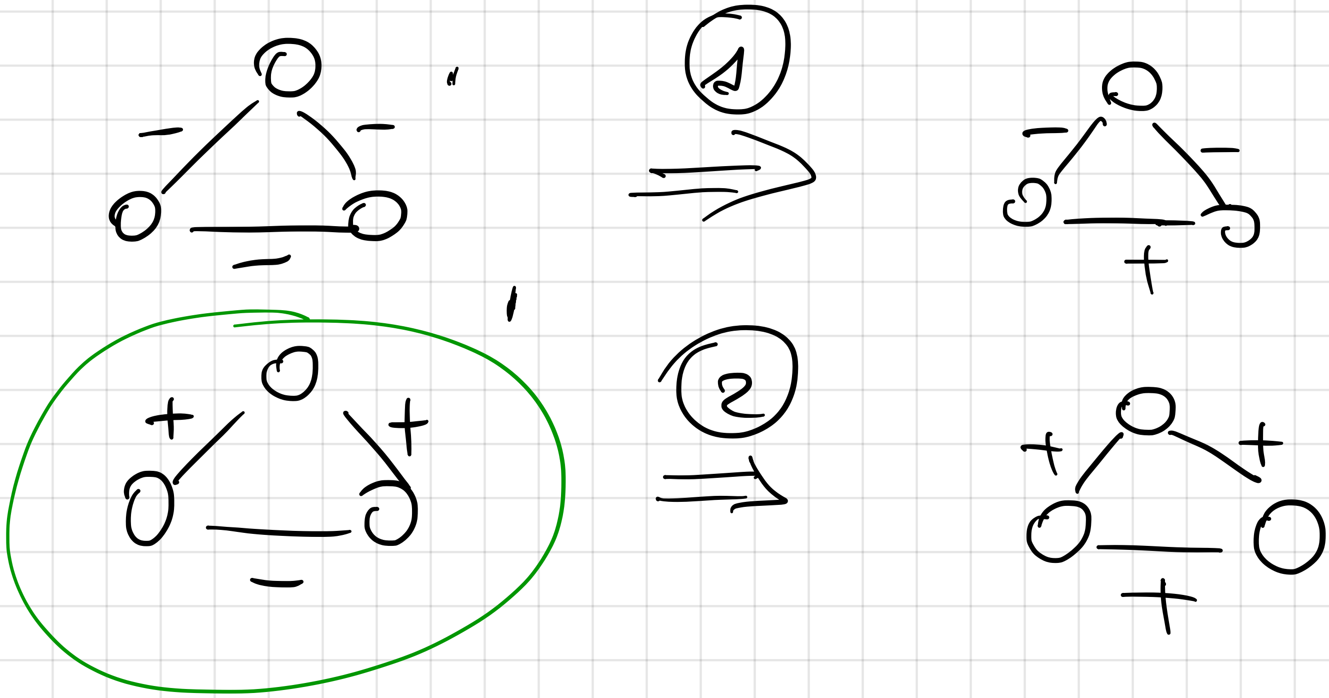
slashdot, ebay,
Epinions, trip Advisor, ...

Guha et al (2004)
"Epinions"
distrust can be
propagated

Leskovec et al. (2010)
"signed networks in
social media"

Balance theory

A weaker form of Structural Balance

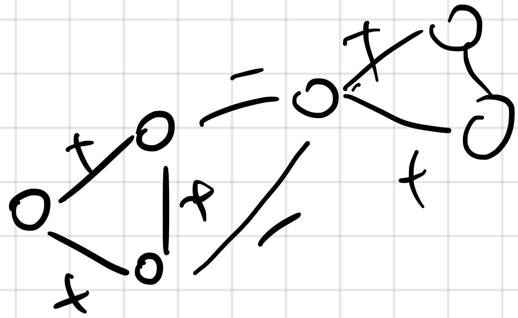


2 is more likely to happen than the 1

(P2) there is no triangle in the graph s.t. we have two positive edges and one negative edge

Weakly Balanced Networks theorem

complete graph & P2



more fragmented decomposition

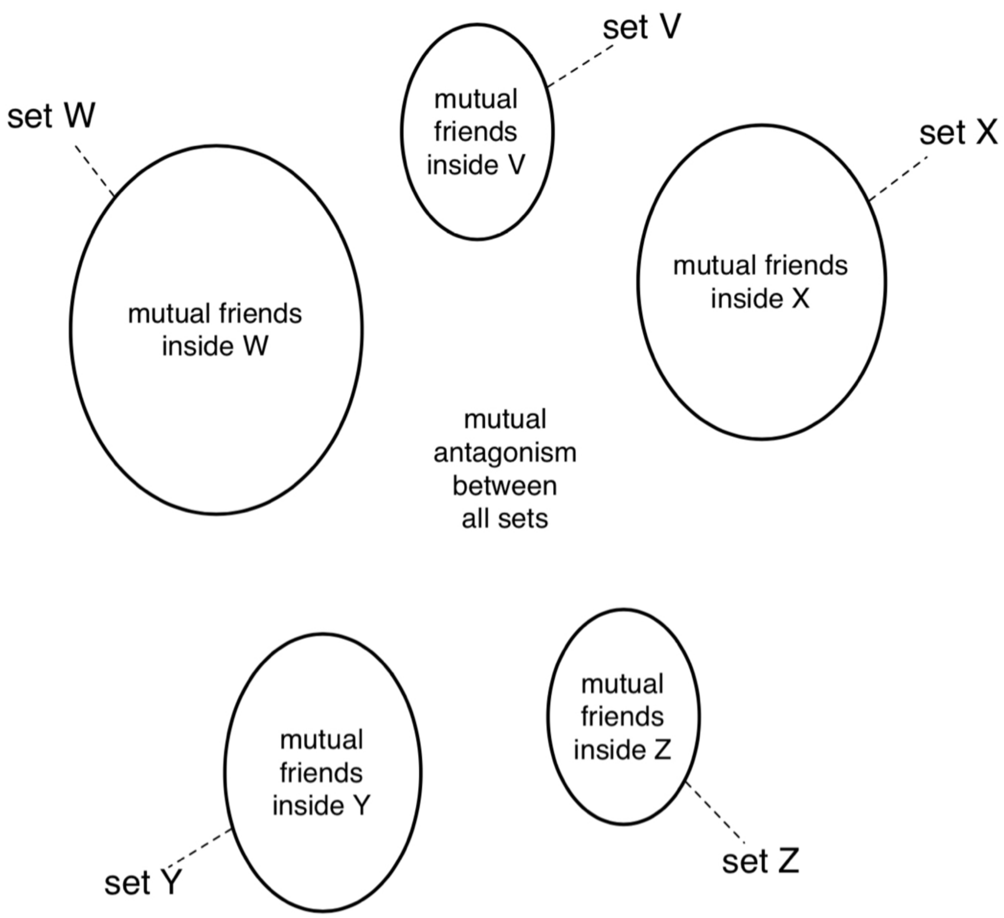


Figure 5.6: A complete graph is weakly balanced precisely when it can be divided into multiple sets mutual friends, with complete mutual antagonism between each pair of sets.

Proof Hp: • complete graph (labeled)
• balanced according P_2

we want to prove that, from A's perspective:

(i) in X every pair of nodes have a positive relationship

(ii) in $X + \{A\}$, nodes are enemies with everyone else

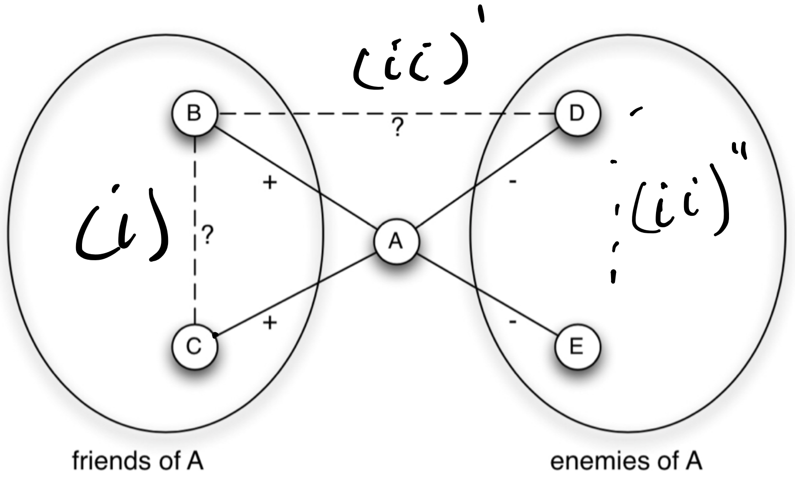


Figure 5.7: A schematic illustration of our analysis of weakly balanced networks. (There may be other nodes not illustrated here.)

(i): $(A, C): +$
$(A, B): +$ $\Rightarrow$ same group $(B, C): +$

(ii)': $(A, B): +$
$(A, D): -$ $\Rightarrow$ $(B, D): -$

(ii)": (A, D) : $-$

(A, E) : $-$

(B, D) : $-$

$\overline{}$

if (D, E) : $+$: in the same group friends each other but enemies with A and $n \in X$

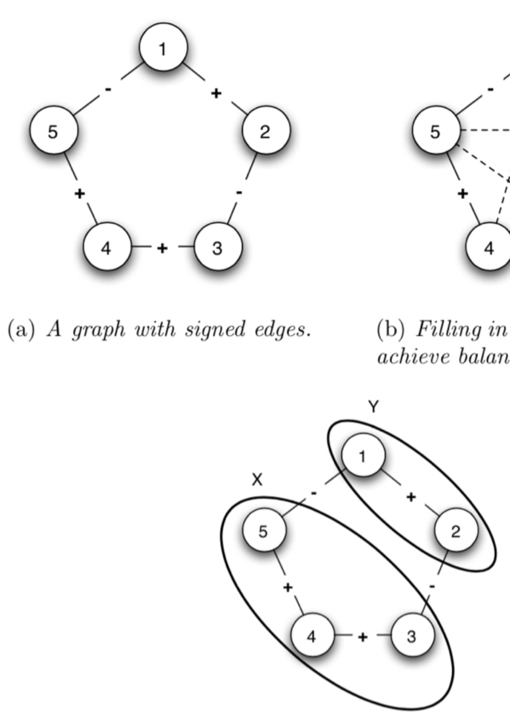
if (D, E) : $-$ they will be in two different groups. $\square$

Generalizing the definition of Structural Balance

Let's relax the "complete graph" assumption

Structural Balance in arbitrary noncomplete graphs

Two approaches:



(a) *A graph with signed edges.* (b) *Filling in the missing edges to achieve balance.*

(c) *Dividing the graph into two sets.*

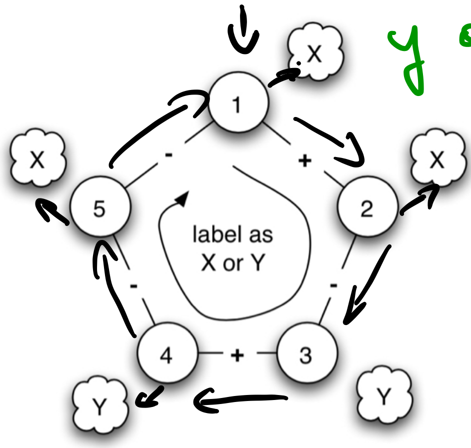
Figure 5.9: There are two equivalent ways to define structural balance for general (non-complete) graphs. One definition asks whether it is possible to fill in the remaining edges so as to produce a signed complete graph that is balanced. The other definition asks whether it is possible to divide the nodes into two sets X and Y so that all edges inside X and inside Y are positive, and all edges between X and Y are negative.

these two definitions are equivalent

Characterizing Balance for General Nets

Harary: the proof contains a method to check balance

start



y at the end!

not balanced

Figure 5.10: If a signed graph contains a cycle with an odd number of negative edges, then it is not balanced. Indeed, if we pick one of the nodes and try to place it in X, then following the set of friend/enemy relations around the cycle will produce a conflict by the time we get to the starting node.

the cycle proves there is no way of dividing in X and Y

observation: we have a cycle with an odd number of "–"

Example

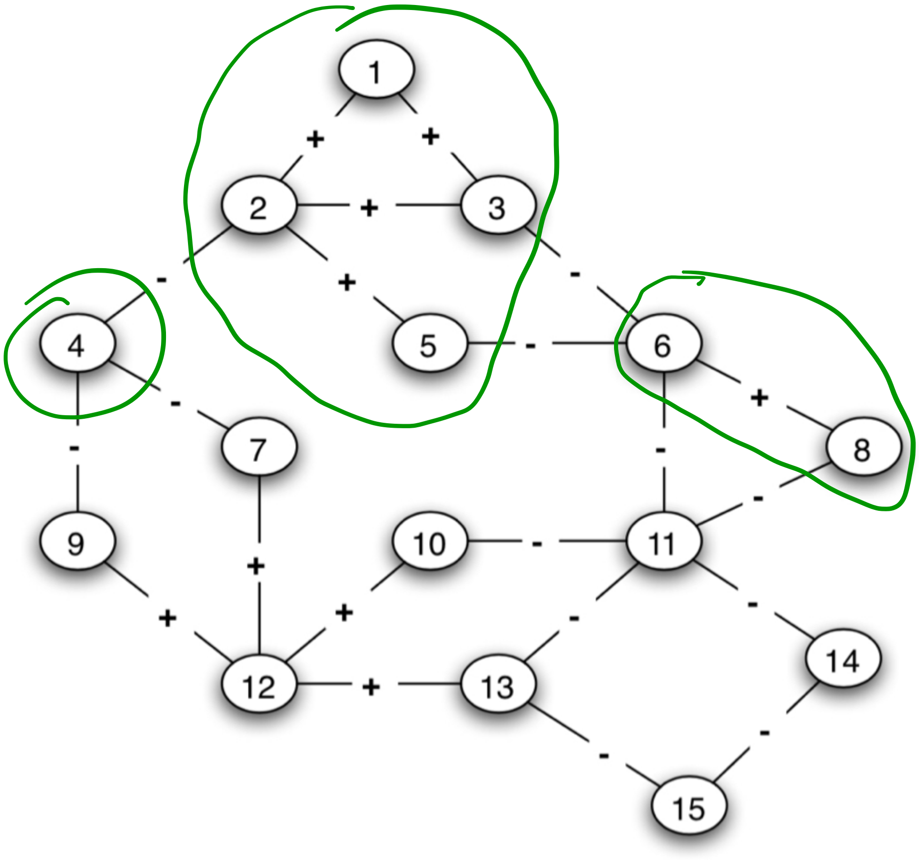


Figure 5.8: In graphs that are not complete, we can still define notions of structural balance when the edges that are present have positive or negative signs indicating friend or enemy relations.

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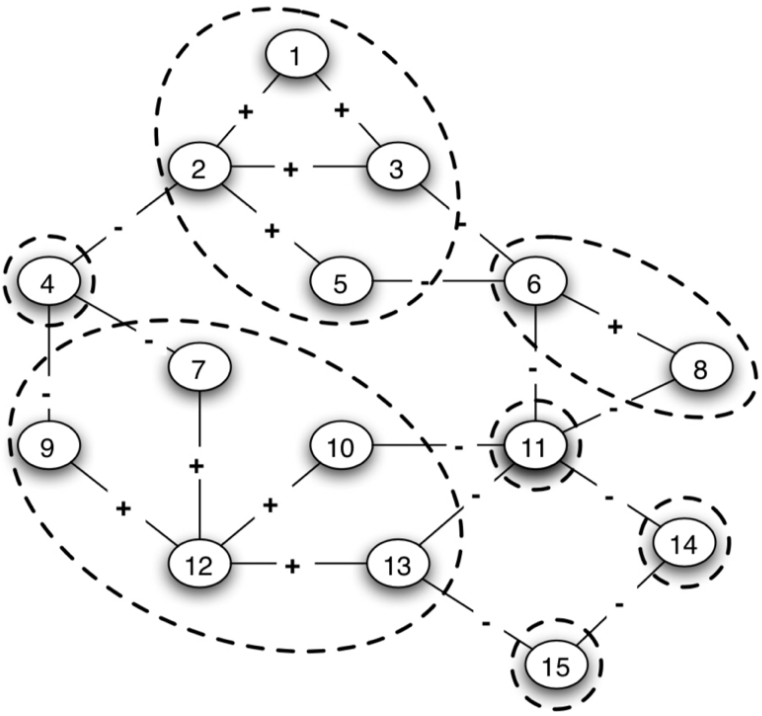


Figure 5.11: To determine if a signed graph is balanced, the first step is to consider only the positive edges, find the connected components using just these edges, and declare each of these components to be a *supernode*. In any balanced division of the graph into X and Y, all nodes in the same supernode will have to go into the same set.

claim: A signed graph is balanced iff it contains no cycle with an odd number of negative edges

Proof we look for a balanced division between two sets X and Y

procedure

1) to convert the graph to a reduced form with only negative edges

"block modeling"

2) to solve the problem in the reduced graph

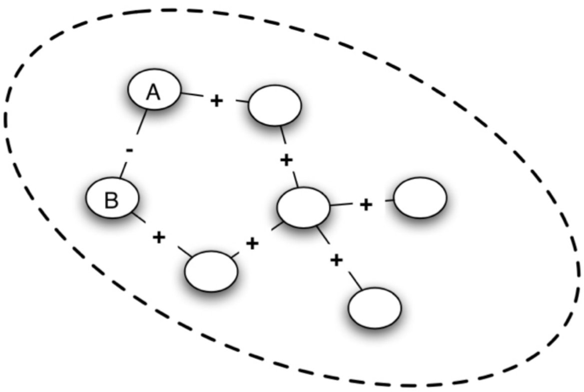


Figure 5.12: Suppose a negative edge connects two nodes A and B that belong to the same supernode. Since there is also a path consisting entirely of positive edges that connects A and B through the inside of the supernode, putting this negative edge together with the all-positive path produces a cycle with an odd number of negative edges.

(Cont'd)

we collapse groups in supernodes

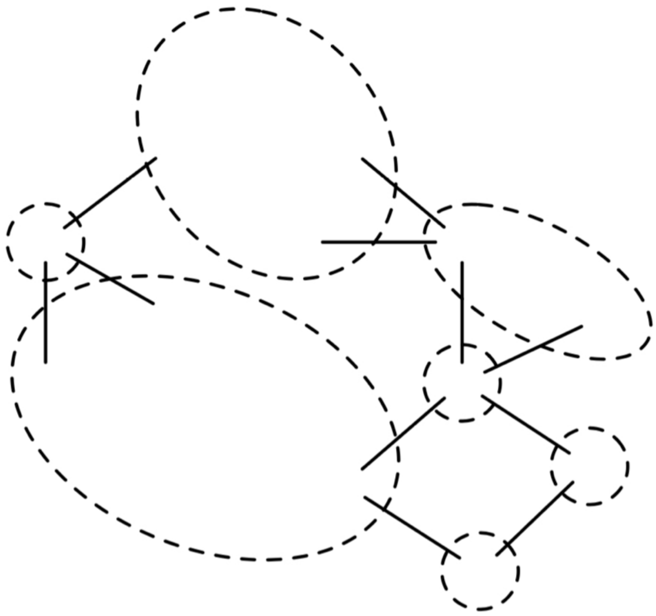


Figure 5.13: The second step in determining whether a signed graph is balanced is to look for a labeling of the supernodes so that adjacent supernodes (which necessarily contain mutual enemies) get opposite labels. For this purpose, we can ignore the original nodes of the graph and consider a *reduced graph* whose nodes are the supernodes of the original graph.

reduced graph

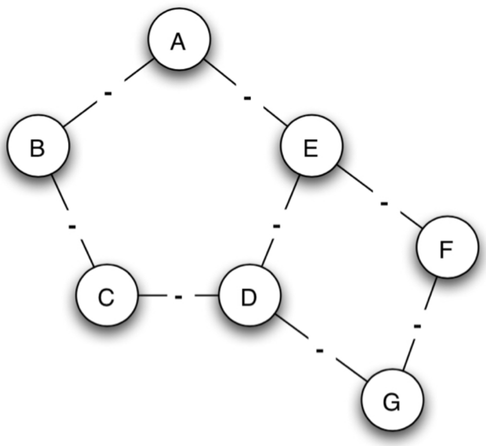


Figure 5.14: A more standard drawing of the reduced graph from the previous figure. A negative cycle is visually apparent in this drawing.

(cont'd)

reduced graphs have only negative edges

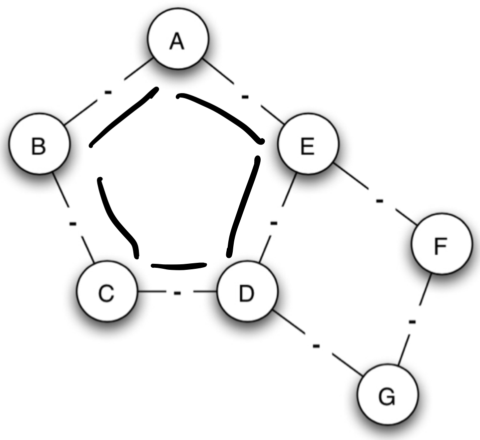
Let's label each node with X or Y

1. if this division is possible
$\Rightarrow$ <u>balanced division</u>

or

2. Look for a cycle with an odd number of negative signs
$\Rightarrow$ we can find such a cycle even in the original graph

proof later



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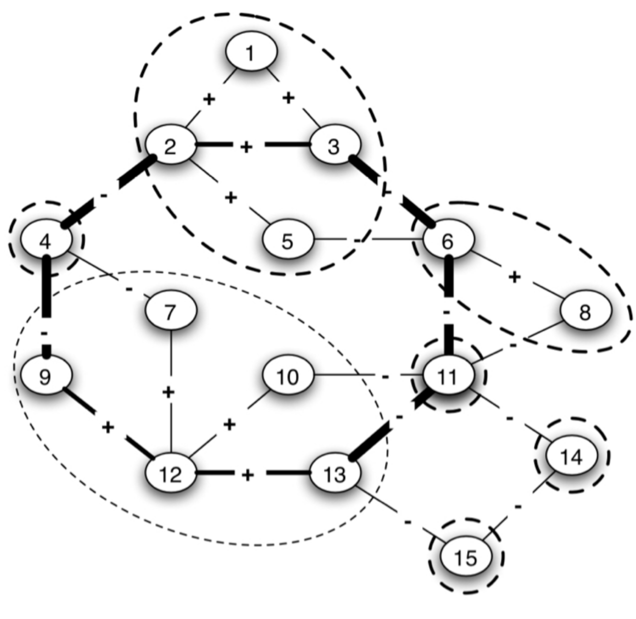


Figure 5.15: Having found a negative cycle through the supernodes, we can then turn this into a cycle in the original graph by filling in paths of positive edges through the inside of the supernodes. The resulting cycle has an odd number of negative edges.

in literature : this is aquivalent to find if the graph is <u>bipartite</u>

(cont'd)

BFS

⇒ output: to produce different layers.

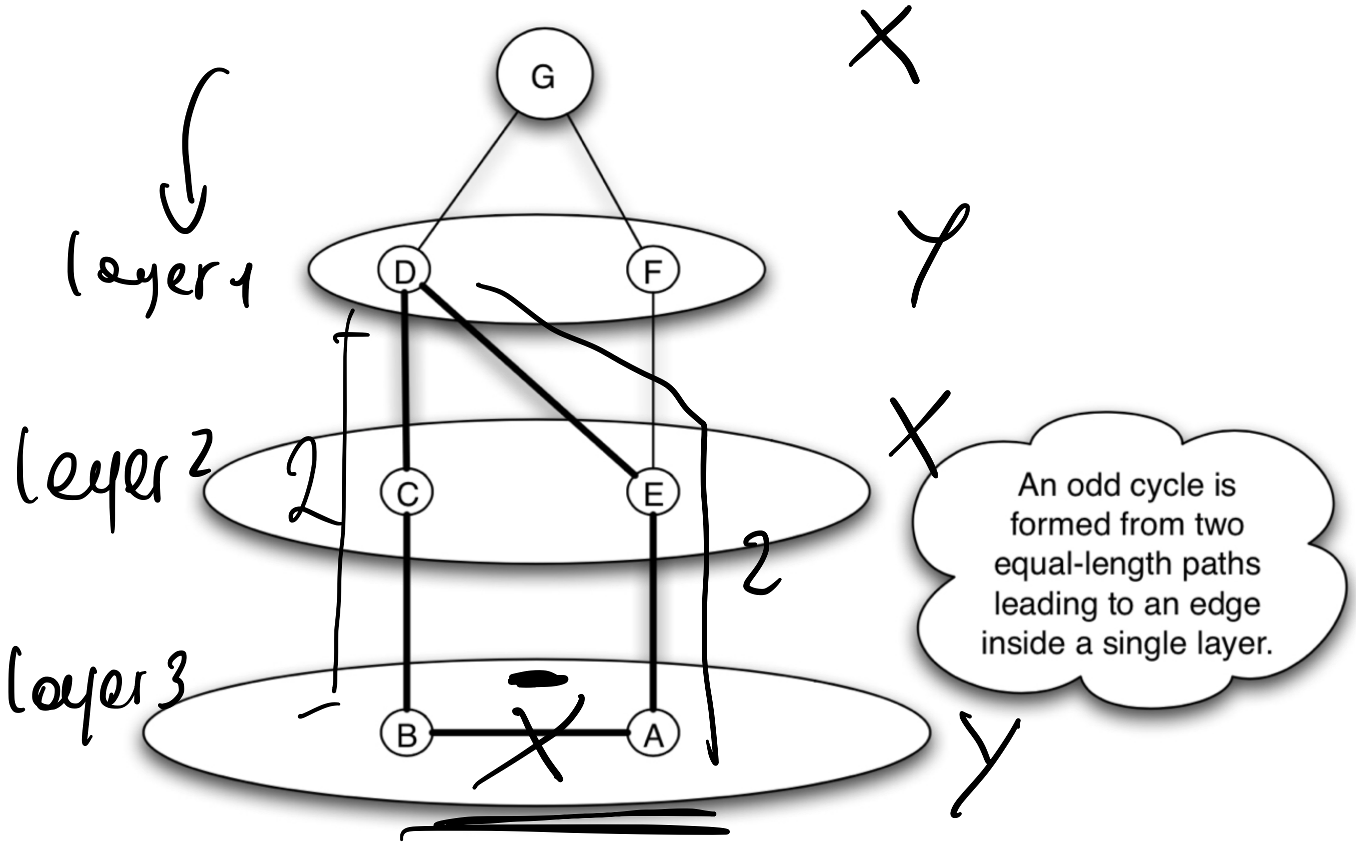
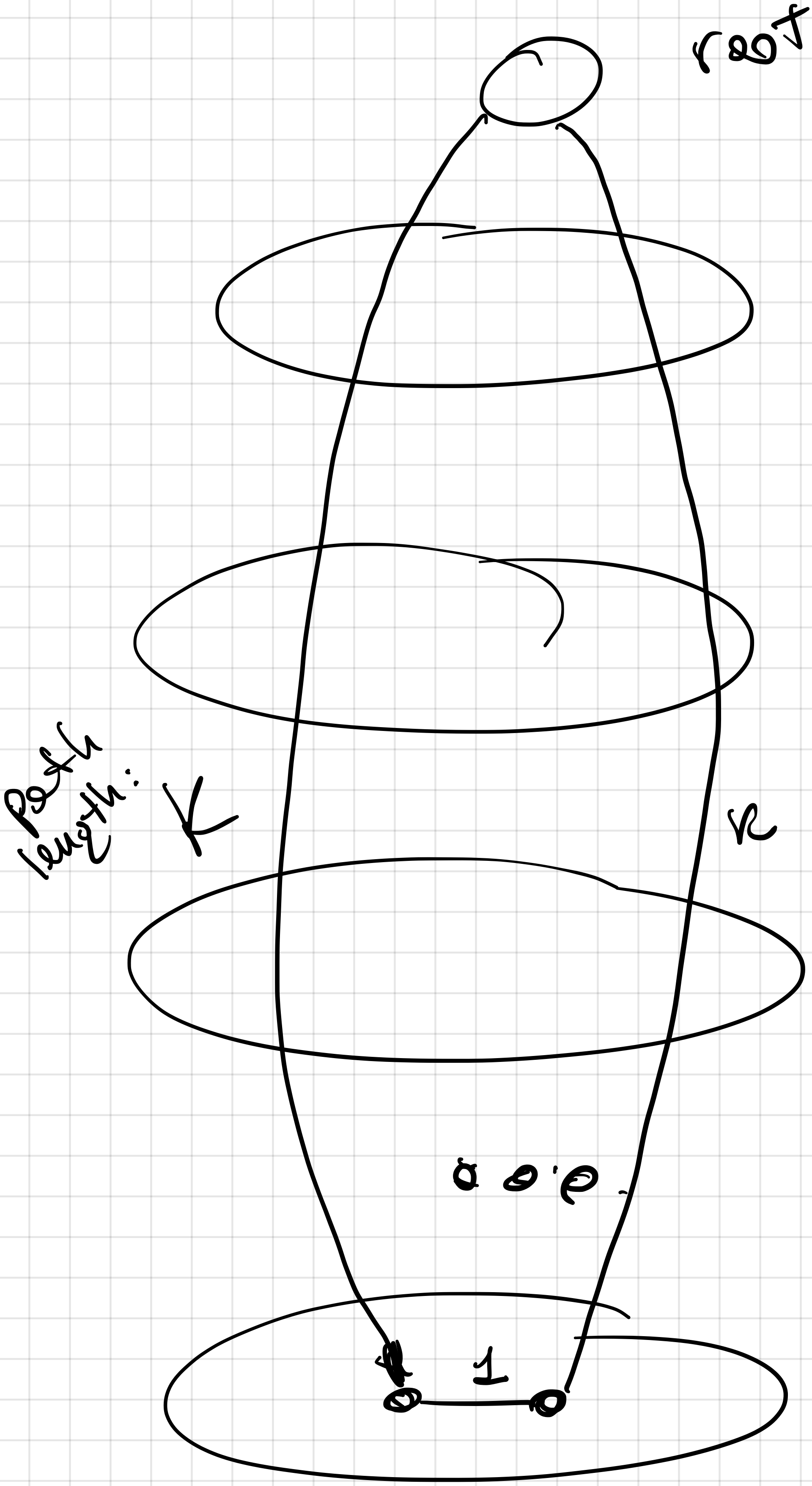


Figure 5.16: When we perform a breadth-first search of the reduced graph, there is either an edge connecting two nodes in the same layer or there isn't. If there isn't, then we can produce the desired division into X and Y by putting alternate layers in different sets. If there is such an edge (such as the edge joining A and B in the figure), then we can take two paths of the same length leading to the two ends of the edge, which together with the edge itself forms an odd cycle.

If we do not find such an edge: we have found a balanced division!

all edges here are negative!



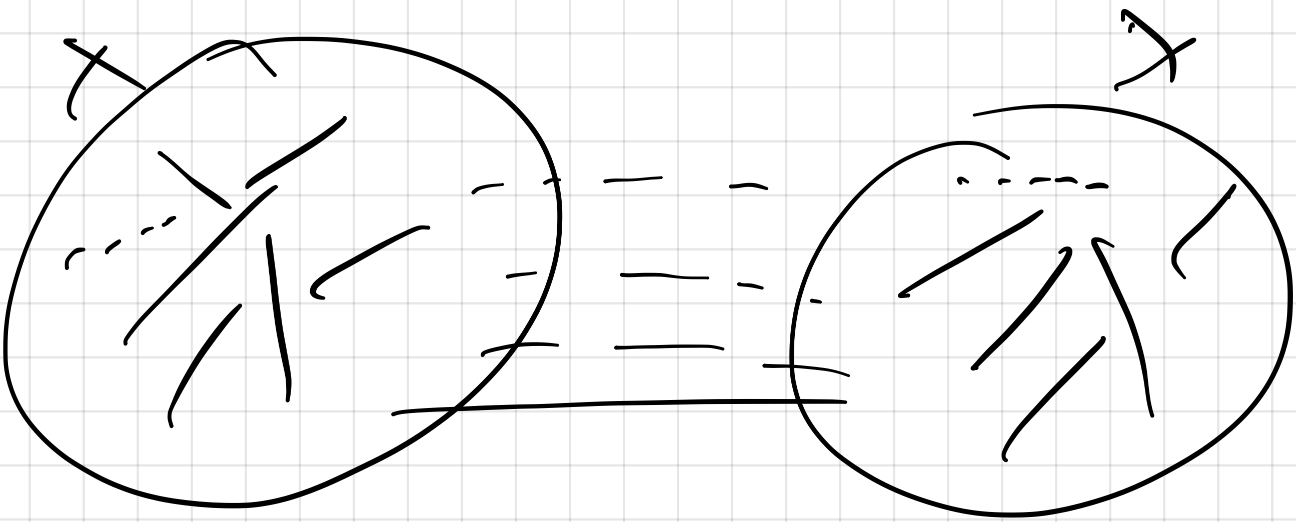
$\underbrace{2K+1}$

odd

K layer

Approximately Balanced Networks

- complete labeled graphs
- weaker definitions of balance
- we look for groups of nodes where only a fraction (majority) of edges are positive



Read it in the book!

Take Home Message

if we have signed networks we can study how to apply Balance theory

to find:

- if there is some tendency toward "balance"
- some explanations of conflicts between different factions
- relationships dynamics

We presented proofs containing methods to check balance (i.e., look for cycles with an odd number of negative edges)

Application to social media: a lot to do!

We also need more validation