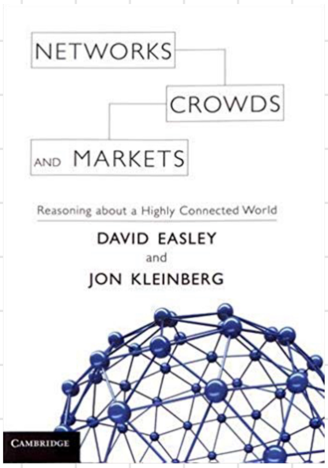
Lecture 16

Network Science

Games

Today's Topics

1. What is a Game
2. Reasoning about Behavior in a Game
3. Best Responses and Dominant Strategies
4. Nash Equilibrium



Chapter 6
6.1 - 6.4
"Games"

Games

we studied "connectedness";
complex systems characterized
"intrinsic interpendence"
=> focus behaviors of individuals

Game theory: decision makers to interact with each other.
Satisfaction does not depend only on individual's choices;

Example

two students, they want to an exam

assumption: they cannot study AND prepare the presentation;
these two students cannot communicate to each other.

Exam: if you study → 92
if you don't study → 80

Presentation : · if you or your partner prepare the pres. : 92 for both

· if neither of you prepare it : 84

· if both of them will prepare the present. : 100

	P	E
P	90, 90	86, 92
E	92, 86	88, 88

player 1

player 2

payoff matrix → it describes the set up

Basic Ingredients

1. Players
2. Strategy : set of options for each player
3. payoff : the outcome for each selected strategy.

$\Rightarrow$ payoff matrix

we want to reason about how two players will behave in a given game

Reasoning about Behavior in a Game

We need a tractable problem.

a. everything that a player cares of is in the payoff matrix.

b. Everything about the structure of the game is known

c. players are rational.

Exam or Presentation Game

		Your Partner: Presentation	Your Partner: Exam
You	Presentation	90, 90	86, 92
	Exam	92, 86	88, 88

Figure 6.1: Exam or Presentation?

your point of view : red

your opponent point of view : blue

strictly dominant strategy : (E)

(E, E)

Counterintuitive : (P, P) would be better for them.

BUT

if you decide to prepare P your partner would be tempted anyhow by her dominant strategy (she/he would get 92)

Prisoner's Dilemme (1950s)

- Police and Two suspects
- no evidence
- suspects are asked to confess.

		Suspect 2 NC	Suspect 2 C
Suspect 1	NC	$-1, -1$	$-10, 0$
	C	$0, -10$	$-4, -4$

Figure 6.2: Prisoner's Dilemma

strictly dominant strategy : (C, C)

Athletes "Drugs" problem

in the short term: better performance

in the long term: you can be caught or you can suffer of health issues.

		Athlete 2	
		Don't Use Drugs	*Use Drugs*
Athlete 1	*Don't Use Drugs*	3, 3	1, 4
	Use Drugs	4, 1	2, 2

Figure 6.3: Performance-Enhancing Drugs

"Use drugs" is strictly dominant strategy for both of them.

Wrapping up the prisoner's dilemma

it arises only when payoffs are designed in a certain way.

simple changes ⇒ more benign outcomes.

an easier exam: you will get 96 if you don't study.

		Your Partner	
		Presentation	*Exam*
You	*Presentation*	98, 98	94, 96
	Exam	96, 94	92, 92

Figure 6.4: Exam-or-Presentation Game with an easier exam.

P: is now the strictly dominant strategy

Best Responses and Dominant Strategies

Tlore for motivation

Players: 1, 2

Strategies: S, T

$P_1(S,T)$: payoff of playing str. S for player 1 and T for player 2.

S: Best Response for player 1

$\forall S'$: $P_1(S,T) \geq P_1(S',T)$

↑ we can have more than one B. R.

S: Strict Best Response

$\forall S'$: $P_1(S,T) > P_1(S',T)$

(symmetrical for player 2 definitions)

Dominant Strategy: a strategy that is best to every strategy of player 2.

$$\forall S', \forall T': \quad P_1(S, T) \geq P_1(S', T')$$

Strict Dominant Strategy: a strategy that is a strict best response to every strategy of player 2

$$\forall S', \forall T': \quad P_1(S, T) > P_1(S', T')$$

What if only one player has a strictly dominant strategy?

two firms planning to produce and market a new product

Two segments of the market:

- people who would buy only a low-priced version of the product (60%)
- people who would buy a upscale version (40%)

One firm: more powerful: Firm 1 reaches 80% of the sales.
Firm 2: will get 20% of the sales.

		Firm 2	
		Low-Priced	*Upscale*
Firm 1	*Low-Priced*	.48, .12	.60, .40
	Upscale	.40, .60	.32, .08

Figure 6.5: Marketing Strategy

Low Prices: strictly dominant strategy for Firm 1

Firm 1 has a s.d.s.: "low-priced"

Players must move simultaneously

Firm 1 can decide its strategy no regards of Firm 2's move

"secrecy"

full knowledge ⟺ payoff matrix

Firm 2 is subordinate to Firm 1:
its best strategy is to stay away from Firm 1 market segment

Nash Equilibrium

What if none has a (strictly) dominant strategy?

=> we need other way to predict what is likely to happen

Example: a three-clients game.

- Two players (Two Firms)
- three large clients: A, B, C
- Three strategies: A, B, C.

If they approach the same client => 50% of the general business.

Firm 1 too small: if it approaches a client on its own: payoff = 0.

A is larger than B and C: it wants to do business with both of them (or nothing)

Firm 2

Firm 1	A	B	C
A	4, 4	0, 2	0, 2
B	0, 0	1, 1	0, 2
C	0, 0	0, 2	1, 1

Figure 6.6: Three-Client Game

A worths 8

B and C worth 2.

dominant strategies : none

(A, A) ⇒ Nash equilibrium

[no players have incentive to change their strategy.]

the system got an equilibrium state, with no force pushing it toward a different configuration

(S, T) is a Nash Equilibrium if

S is a best response to T

T is a best response to S

Take home messages

1. Game theory gives us a "simplified" framework to understand how individual strategies can create an intrinsic interdependence in the behaviors of participants to a complex systems

2. Basic Terms: players, strategies, payoffs

3. We may have (strictly) best responses to other players moves and (strictly) dominant strategies

4. When no players have (strictly) dominant strategies we can still look for Nash Equilibria: a set of strategies s.t. no player has incentive to deviate from.