



GPU Teaching Kit
Accelerated Computing



Module 4.5 - Memory and Data Locality

Handling Arbitrary Matrix Sizes in Tiled Algorithms

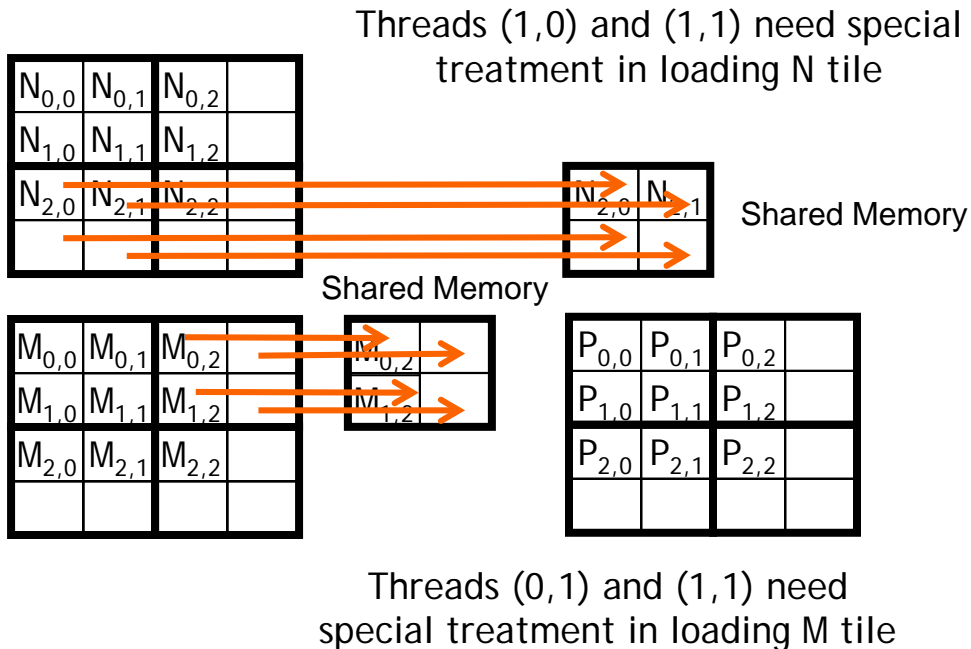
Objective

- To learn to handle arbitrary matrix sizes in tiled matrix multiplication
 - Boundary condition checking
 - Regularizing tile contents
 - Rectangular matrices

Handling Matrix of Arbitrary Size

- The tiled matrix multiplication kernel we presented so far can handle only square matrices whose dimensions (Width) are multiples of the tile width (TILE_WIDTH)
 - However, real applications need to handle arbitrary sized matrices.
 - One could pad (add elements to) the rows and columns into multiples of the tile size, but would have significant space and data transfer time overhead.
- We will take a different approach.

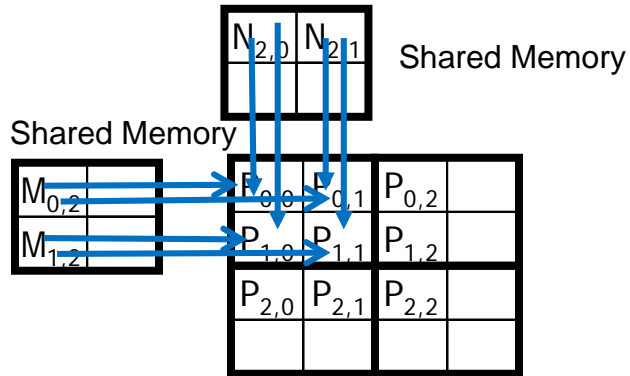
Phase 1 Loads for Block (0,0) for a 3x3 Example



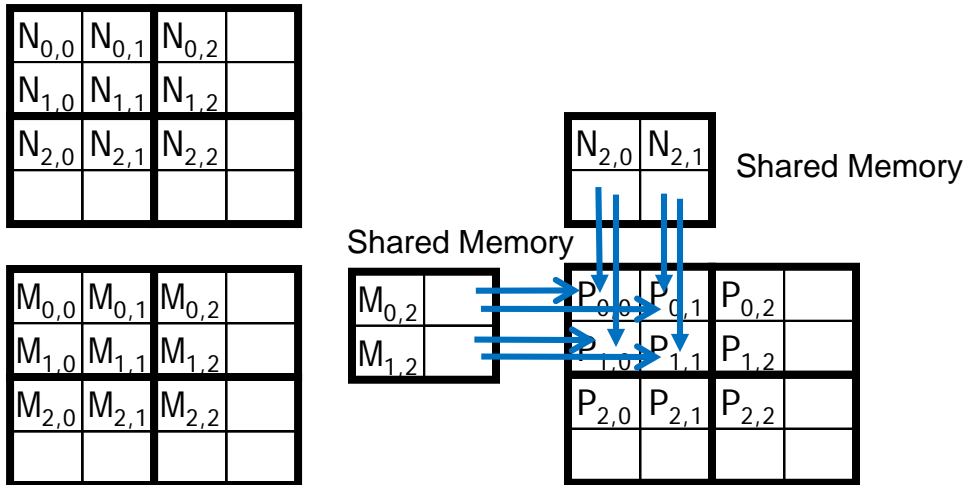
Phase 1 Use for Block (0,0) (iteration 0)

$N_{0,0}$	$N_{0,1}$	$N_{0,2}$	
$N_{1,0}$	$N_{1,1}$	$N_{1,2}$	
$N_{2,0}$	$N_{2,1}$	$N_{2,2}$	

$M_{0,0}$	$M_{0,1}$	$M_{0,2}$	
$M_{1,0}$	$M_{1,1}$	$M_{1,2}$	
$M_{2,0}$	$M_{2,1}$	$M_{2,2}$	



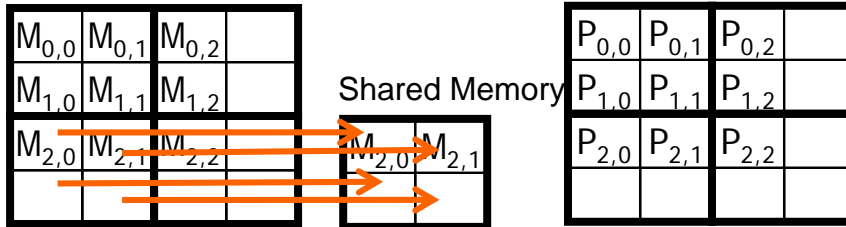
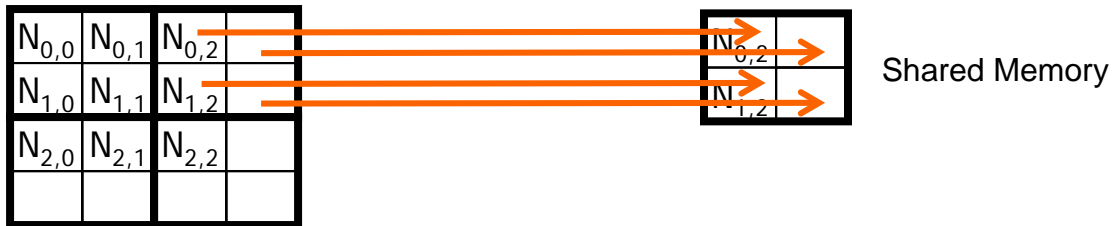
Phase 1 Use for Block (0,0) (iteration 1)



All Threads need special treatment. None of them should introduce invalidate contributions to their P elements.

Phase 0 Loads for Block (1,1) for a 3x3 Example

Threads (0,1) and (1,1) need special treatment in loading N tile



Threads (1,0) and (1,1) need special treatment in loading M tile

Major Cases in Toy Example

- Threads that do not calculate valid P elements but still need to participate in loading the input tiles
 - Phase 0 of Block(1,1), Thread(1,0), assigned to calculate non-existent P[3,2] but need to participate in loading tile element N[1,2]
- Threads that calculate valid P elements may attempt to load non-existing input elements when loading input tiles
 - Phase 0 of Block(0,0), Thread(1,0), assigned to calculate valid P[1,0] but attempts to load non-existing N[3,0]

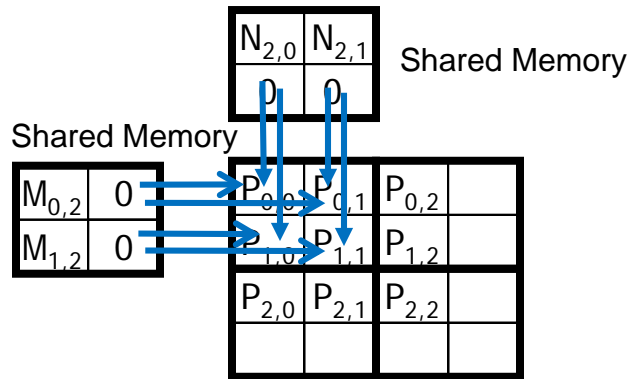
A “Simple” Solution

- When a thread is to load any input element, test if it is in the valid index range
 - If valid, proceed to load
 - Else, do not load, just write a 0
- Rationale: a 0 value will ensure that that the multiply-add step does not affect the final value of the output element
- The condition tested for loading input elements is different from the test for calculating output P element
 - A thread that does not calculate valid P element can still participate in loading input tile elements

Phase 1 Use for Block (0,0) (iteration 1)

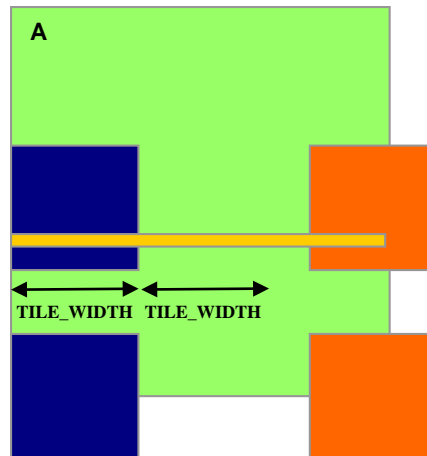
$N_{0,0}$	$N_{0,1}$	$N_{0,2}$	
$N_{1,0}$	$N_{1,1}$	$N_{1,2}$	
$N_{2,0}$	$N_{2,1}$	$N_{2,2}$	

$M_{0,0}$	$M_{0,1}$	$M_{0,2}$	
$M_{1,0}$	$M_{1,1}$	$M_{1,2}$	
$M_{2,0}$	$M_{2,1}$	$M_{2,2}$	



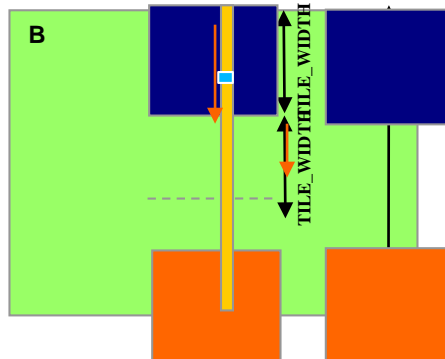
Boundary Condition for Input M Tile

- Each thread loads
 - $M[\text{Row}][p \cdot \text{TILE_WIDTH} + tx]$
 - $M[\text{Row} \cdot \text{Width} + p \cdot \text{TILE_WIDTH} + tx]$
- Need to test
 - $(\text{Row} < \text{Width}) \ \&\& \ (p \cdot \text{TILE_WIDTH} + tx < \text{Width})$
 - If true, load M element
 - Else , load 0



Boundary Condition for Input N Tile

- Each thread loads
 - $N[p * \text{TILE_WIDTH} + ty][\text{Col}]$
 - $N[(p * \text{TILE_WIDTH} + ty) * \text{Width} + \text{Col}]$
- Need to test
 - $(p * \text{TILE_WIDTH} + ty < \text{Width}) \ \&\& \ (\text{Col} < \text{Width})$
 - If true, load N element
 - Else , load 0



Loading Elements – with boundary check

```
- 8  for (int p = 0; p < (Width-1) / TILE_WIDTH + 1; ++p) {  
-  
-  ++    if (Row < Width && t * TILE_WIDTH + tx < Width) {  
- 9      ds_M[ty][tx] = M[Row * Width + p * TILE_WIDTH + tx];  
-  ++    } else {  
-  ++      ds_M[ty][tx] = 0.0;  
-  ++    }  
-  ++    if (p * TILE_WIDTH + ty < Width && Col < Width) {  
- 10     ds_N[ty][tx] = N[(p * TILE_WIDTH + ty) * Width + Col];  
-  ++    } else {  
-  ++      ds_N[ty][tx] = 0.0;  
-  ++    }  
- 11    __syncthreads();  
-  
-
```

Inner Product – Before and After

```
- ++ if(Row < Width && Col < Width) {  
- 12   for (int i = 0; i < TILE_WIDTH; ++i) {  
- 13       Pvalue += ds_M[ty][i] * ds_N[i][tx];  
-     }  
- 14   __syncthreads();  
- 15 } /* end of outer for loop */  
- ++ if (Row < Width && Col < Width)  
- 16     P[Row*Width + Col] = Pvalue;  
- } /* end of kernel */
```

Some Important Points

- For each thread the conditions are different for
 - Loading M element
 - Loading N element
 - Calculating and storing output elements
- The effect of control divergence should be small for large matrices

Handling General Rectangular Matrices

- In general, the matrix multiplication is defined in terms of rectangular matrices
 - A $j \times k$ M matrix multiplied with a $k \times l$ N matrix results in a $j \times l$ P matrix
- We have presented square matrix multiplication, a special case
- The kernel function needs to be generalized to handle general rectangular matrices
 - The Width argument is replaced by three arguments: j , k , l
 - When Width is used to refer to the height of M or height of P , replace it with j
 - When Width is used to refer to the width of M or height of N , replace it with k
 - When Width is used to refer to the width of N or width of P , replace it with l



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