Lecture 17

Network Science

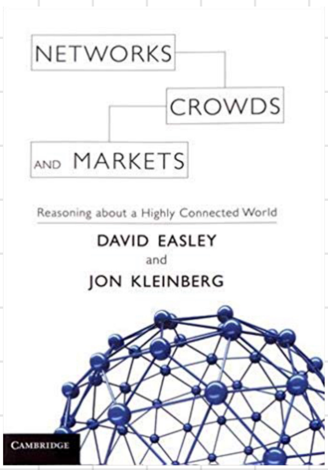
Mixed Strategies and Multiple Equilibria

Today's Topics

1. Multiple Equilibria
   - Coordination Games
   - the Hawk-Dove Game

2. Mixed Strategies
   - Example and Empirical Analysis

3. Pareto Optimality and Social Optimality



Chapter 6

6.5 - 6.9

"Games"

Recap: Nash Equilibrium

		Firm 2		
		A	B	C
Firm 1	A	4, 4	0, 2	0, 2
	B	0, 0	1, 1	0, 2
	C	0, 0	0, 2	1, 1

Figure 6.6: Three-Client Game

(A, A) is a Nash Equilibrium

player 1: selects strategy S
player 2: " " T

(S, T) is a N.E. if
S is the best response to T
and T is the best response to S

Equilibrium:

- it cannot be derived from pure "rationality" (not always a dominant strategy)
- it's a system state where no force is pushing toward a different outcome

No player has an incentive to change its strategy.

Multiple Equilibria: Coordination Games

what if we have more than one N.E.?

you and your partner : players
you need to prepare a joint present.
which software?
Apple Keynote or Microsoft Power Point?
You need to "coordinate" (no communication)

		Your Partner	
		PowerPoint	*Keynote*
You	*PowerPoint*	1, 1	0, 0
	Keynote	0, 0	1, 1

Figure 6.7: Coordination Game

(P, P) & (K, K) are both N.E.

<u>what to do?</u>

thomas Schelling's idea of
Focal Point: natural reasons
to focus on one of the N.E.

⇒ (Social) conventions outside
the pay-off matrix can
help.

For example, you and your partner have a preference on software. You can just change your payoff matrix

		Your Partner	
		PowerPoint	*Keynote*
You	*PowerPoint*	1, 1	0, 0
	Keynote	0, 0	2, 2

Figure 6.8: Unbalanced Coordination Game

Schelling's idea of focal points: try to embed in your payoff matrix the intrinsic features that help you to select an equilibrium

The Battle of the Sexes

What to do if players do not agree?

		Your Partner	
		PowerPoint	*Keynote*
You	*PowerPoint*	1, 2	0, 0
	Keynote	0, 0	2, 1

Figure 6.9: Battle of the Sexes

It is hard to predict the equilibrium that will be played

We need some other a priori agreement

the Stag - Hunt Game

two hunters

hunt together: they can catch a stag

hunt seperated: they will catch a hare each

but if one tries to be cooperative, it loses.

		Hunter 2	
		Hunt Stag	*Hunt Hare*
Hunter 1	*Hunt Stag*	4, 4	0, 3
	Hunt Hare	3, 0	3, 3

N.E.

Figure 6.10: Stag Hunt

if one is cooperative, and the other is not, the "selfish" hunter will get a hare the other will get nothing.

Somehow similar to the prisoner's dilemma: if they coordinate, they will get the highest payoff, but trying to coordinate is highly risky.

Multiple Equilibria: the Hawk-Dove Game

"anti coordination game"

two animals

they can decide < Hawk (aggressive) / Dove (passive)

both passive: each gets 3

both aggressive: each gets 0

if one is aggressive: gets 5 (the other: 1)

		Animal 2	
		D	H
Animal 1	D	3, 3	1, 5
	H	5, 1	0, 0

Figure 6.12: Hawk-Dove Game

N.E.: (H, D), (D, H)

without other knowledge we cannot predict which of these equilibria will be played.

Mixed Strategies

with No N.E. at all:

Let's introduce randomization and probabilities

Matching Pennies

Matching Pennies: each player has a penny.

they can show head or tail.

match: player 1 loses

no match: player 1 wins

			Player 2	
			H	T
Player 1	H		$-1,+1$	$+1,-1$
	T		$+1,-1$	$-1,+1$

Figure 6.14: Matching Pennies

(zero sum games)

No Nash Equilibria

(Also called "Attack - Defense" Games)

we model this game with "randomization"

prob. p: player 1 chooses H
(prob. $1-p$, $\Rightarrow$ T)

prob. q: player 2 chooses H
(prob. $1-q$ $\Rightarrow$ T)

strategies are probabilities between $[0, 1]$

mixed strategies: it involves a "mixing" between the options H and T.

if $p = 0$ $\Rightarrow$ P1 is playing T

if $p = 1$ $\Rightarrow$ P1 is playing S

...

" pure strategies "

Payoffs for mixed strategies

payoffs are "random". How to rank them?

player 1's point of view:

p2 will play H with prob. q

p2 〃 〃 T 〃 〃 $1-q$

expected payoff of "pure strategy" H

$$(-1)\cdot q + (+1)(1-q) =$$

$$\underline{1 - 2q}$$

expected payoff of "pure strategy" T

$$+1 \cdot q + (-1)(1-q) =$$

$$\underline{2q - 1}$$

P1 wants to maximize the expected payoff

Equilibrium with Mixed Strategies

- in matching penny : No N. E.
- no "pure strategies" are part of a given N. E.
- strategies $\in]0, 1[$

Key Point :

$$\overbrace{1 - 2q}^{\text{expected payoff of pure strategy } S} \stackrel{?}{=} \overbrace{2q - 1}^{\text{expected payoff of pure strategy } T}$$

if $1 - 2p \neq 2p - 1 \Rightarrow$ then one payoff will be greater than the other; but no "pure strategies"! IMPOSSIBLE

$$\Rightarrow \quad 1 - 2p = 2p - 1$$

$$\Rightarrow \quad q = \frac{1}{2}$$

symmetrically, from p2's perspective

$$p = \frac{1}{2}$$

$\left(p = \frac{1}{2}, q = \frac{1}{2}\right)$ is a N.E. !

Interpretation

"If player 1 believes that player 2 will choose H more than half of the times, then she will win more than half of the times simply choosing T" (symmetric reasoning applies as well)

the choice of $q = \frac{1}{2}$ is "unexploitable" by player 1.

"indifference principle":

the choice of q and p are "unexploitable" for the other player to decide their strategy.

Nash main results: he proved that every game has at least one N.E.

Mixed Strategies: Examples and Empirical Analysis (Sports)

the "Run - Pass" Game (American Football)

Just look at the payoff matrix below:

		Defense: *Defend Pass*	Defense: *Defend Run*
Offense	*Pass*	0, 0	10, −10
	Run	5, −5	0, 0

Figure 6.15: Run-Pass Game

No N. E. with pure strategies

p: prob. for the offense to play "pass"

q: prob. for the defense to defend against the "pass" expected payoff to the offense from passing:

$$0 \cdot q + 10 \cdot (1-q) =$$

$$= 10 - 10q$$

expected payoff to the offense from running

$$5 \cdot q + 0(1-q) = 5q$$

Indifference Principle:

$$10 - 10q = 5q \Rightarrow q = \frac{2}{3}$$

analogous reasoning

$$p = \frac{1}{3}$$

$\left(p = \frac{1}{3}, q = \frac{2}{3}\right)$ is an equilibrium

asymmetric payoff matrix $\Rightarrow$ unbalanced probabilities

Strategic Interpretation of the Run - Pass Game

$p = \frac{1}{3}$: passing is the offense's most powerful weapon, but it is used less than half of the time!

Counterintuitive, but... strategically makes perfect sense!

$q = \frac{2}{3}$: it means that the defense is defending against the pass $\frac{2}{3}$ of the times: somehow the "threat" of passing is helping the offense, even though it uses it relatively rarely!

American Football Statistics: it is possible to verify that teams generally run more than they pass!

The penalty-kick game

soccer

two players game : the kicker and the goalie

two strategies : (kick/dive) Left
(kick/dive) Right

- Based on an analysis performed on 1400 penalty kicks in professional soccer.
- Palacios-Huerta (2002) was able to get the empirical probabilities of kicking left or right and if the goalies jumped left or right; he collected also the final outcomes to create the following payoff matrix :

		Goalie L	Goalie R
Kicker	L	0.58, −0.58	0.95, −0.95
	R	0.93, −0.93	0.70, −0.70

Figure 6.16: The Penalty-Kick Games (from empirical data [337]).

unbalanced probably because there are more right-footed kickers

Let's apply "principle of indifference":

q : prob. for the goalie to choose L

$$.58 \cdot q + (.95) \cdot (1-q)$$

$$\Rightarrow \quad q = .42$$

analogous calculation for p:

$$p = .39$$

- from data drawn from real penalty kicks the goalies dive left $\approx .42$ fraction of the times
- the kickers aim left $.40$ fraction of the times (precision $- 0.01$)

Validated!

Optimalities

- We have Nash Equilibria, s.t. each player's strategy is a best response to the other player's strategy.
- this does not mean that the players will necessarily reach an outcome that is in any sense "good"

- It is possible to classify outcomes not just by their strategic or equilibrium properties, but also by whether they are "good" for ourselves and "the others".

Pareto Optimality

A choice of strategies - one by each players - is Pareto Optimal if there is no other choice of strategies in which all players receive payoffs at least as high, and at least one player receives a strictly higher payoff

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A binding agreement to actually play the "superior" pairs of strategies is usually needed.

N.E. are not Pareto optima

		Your Partner	
		Presentation	*Exam*
You	*Presentation*	90, 90	86, 92
	Exam	92, 86	88, 88

Figure 6.18: Exam or Presentation?

Pareto optimalities

three Pareto optima

players have the incentive to change their strategy, unless they do have a binding agreement

Social Optimality (stronger)

A choice of strategies – one by each players – is a social welfare maximizer (or social optimum) if it maximizes the sum of the players' payoffs

		Your Partner	
		Presentation	*Exam*
You	*Presentation*	90, 90	86, 92
	Exam	92, 86	88, 88

Figure 6.18: Exam or Presentation?

(P,P) social optimum (and also Pareto optimum)

N.E. is not a social optimum

Take Home Messages

1. When we have more than one Nash Equilibria, we need some other agreements

2. In coordination games: focal points can help

3. In anti coordination games: we need some other knowledge to predict which equilibria will be played

4. With no Nash Equilibrium based on "pure strategies", we need to move to randomization and "mixed strategies"

5. Pareto and Social Optimality: some binding agreement can help to get rid of Nash Equilibria and to aim to better welfare.