# 21 Epidemics pp645

21.1 Diseases and the Networks that Transmit Them
21.2 Branching Processes
21.3 The SIR Epidemic Model
21.4 The SIS Epidemic Model
21.5 Synchronization

## Pre-1500 [ edit ]

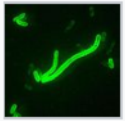
| Death toll (estimate) | Location | Date | Article | Disease | Ref. |
|---|---|---|---|---|---|
| 25–50 million; 40% of population | Europe, Egypt, West Asia | 541–542 | Plague of Justinian | plague | [4] |
| 100,000+ | Ctesiphon, Persia | 627 | | plague | [5] |
| | British Isles | 664–668 | Plague of 664 | plague | [6][page needed] |
| | British Isles | 680–686 | | plague | [6][page needed] |
| | Byzantine Empire, West Asia, Africa | 746–747 | | plague | [7] |
| 75–200 million; 30–60% of population | Europe, Asia and North Africa | 1346–1350 | Black Death | plague  Yersinia pestis | [8] |
| | Europe | 250–266 | Plague of Cyprian | unknown, possibly smallpox | [3] |
| 75,000–100,000 | Greece | 429–426 BC | Plague of Athens | unknown, possibly typhus | [1] |
| 5 million; 30% of population in some areas | Europe, Western Asia, Northern Africa | 165–180 | Antonine Plague | unknown, symptoms similar to smallpox | [2] |

**Table 1. Behavioral Response to 2009 H1N1 Influenza during the Early Months of the Pandemic.***

| Behavior | April 2009 | May 2009 | June 2009 |
|---|---|---|---|
| | | *% of respondents* | |
| **Initial recommendation** | | | |
| Wash your hands or use hand sanitizer more frequently | 59 | 67 | 62 |
| Make preparations to stay at home if you or a family member are sick | Not asked | 55 | Not asked |
| Take steps to avoid being near someone with flulike symptoms† | Not asked | 35 | 38 |
| **Interaction with others** | | | |
| Avoid places where many people are gathered together, such as sporting events, malls, or public transportation‡§ | 25 | 25 | 16 |
| Avoid people you think may have recently visited Mexico | 20 | 16 | Not asked |
| Avoid Mexican restaurants or grocery stores | 17 | 13 | Not asked |
| Reduce contact with people outside your own household as much as possible | Not asked | Not asked | 20 |
| Avoid large shopping areas or malls | Not asked | Not asked | 14 |
| Avoid air travel§ | Not asked | 27 | 13 |
| Limit your use of public transportation, buses, and trains | Not asked | Not asked | 12 |
| Reduce your attendance at church, temple, mosque, or other place of worship | Not asked | Not asked | 6 |
| Avoid family or personal events, such as parties, wedding ceremonies, or funeral services | Not asked | Not asked | 6 |
| **Physical contact with others during interaction** | | | |
| Stop shaking hands with people† | Not asked | 14 | 12 |
| Stop hugging and kissing close friends or relatives† | Not asked | 12 | 9 |
| Wear a face mask | 8 | 4 | 6 |
| **Contact with health professionals** | | | |
| Talk with your doctor about health issues related to H1N1 or swine flu | 5 | 8 | Not asked |
| Get a prescription for or purchase an antiviral drug, such as Tamiflu or Relenza | 1 | 2 | 3 |

* Data are from Harvard School of Public Health polls, April, May, and June 2009.
† The language in this question focused on "you personally" rather than on "you or anyone in your household."
‡ The June 28, 2009, poll did not include "or public transportation."
§ In April and May polls, respondents were provided with "yes" or "no" response options for this question. In June, respondents were given an additional explicit category of "No one in your household did this before the outbreak."

THE Susceptible Infected Recovered (SIR) MODEL

- Initially, some nodes are in the $I$ state and all others are in the $S$ state.
- Each node $v$ that enters the $I$ state remains infectious for a fixed number of steps $t_I$.
- During each of these $t_I$ steps, $v$ has a probability $p$ of passing the disease to each of its susceptible neighbors.
- After $t_I$ steps, node $v$ is no longer infectious or susceptible to further bouts of the disease; we describe it as *removed* ($R$), since it is now an inert node in the contact network that can no longer either catch or transmit the disease.

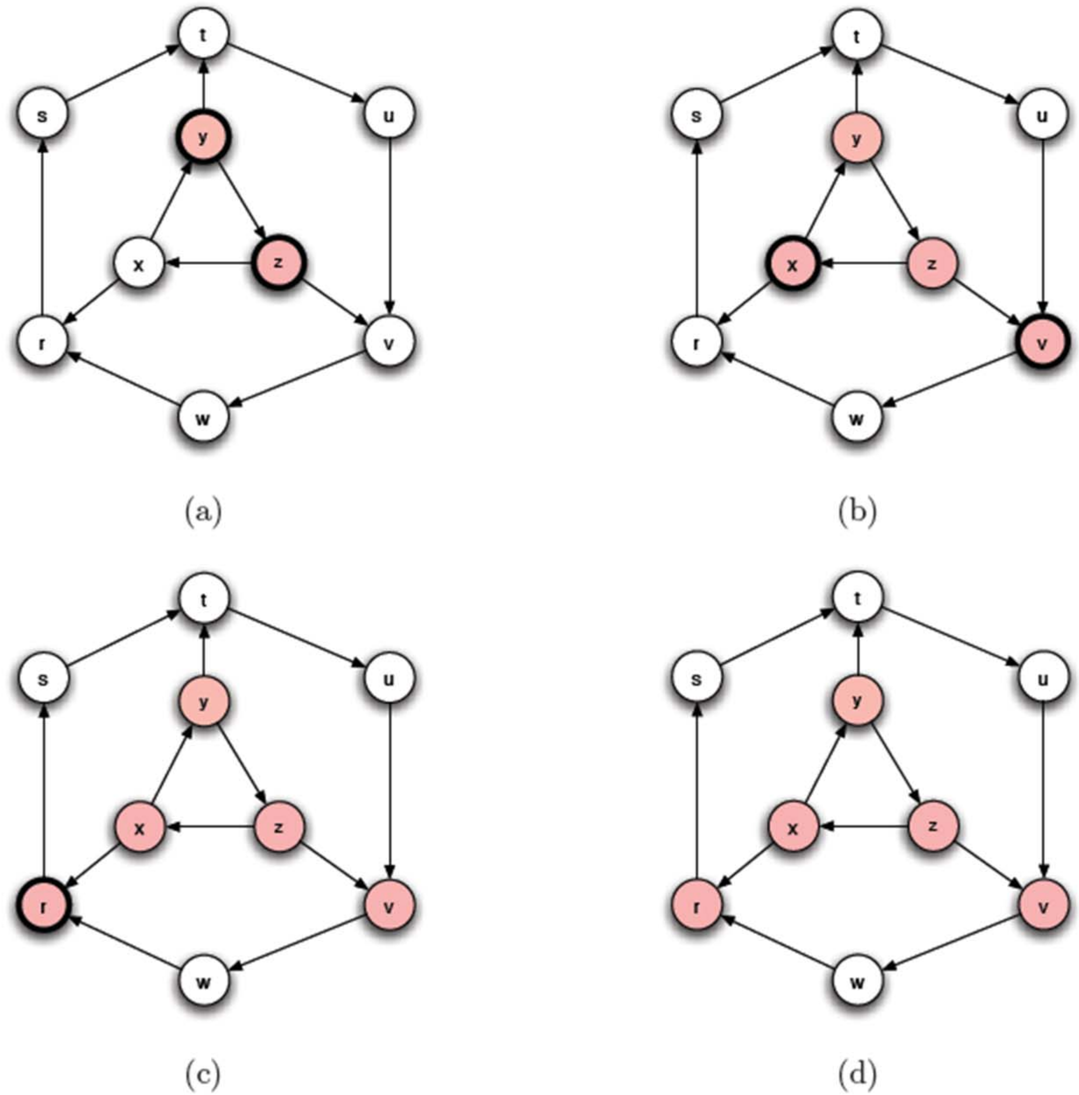
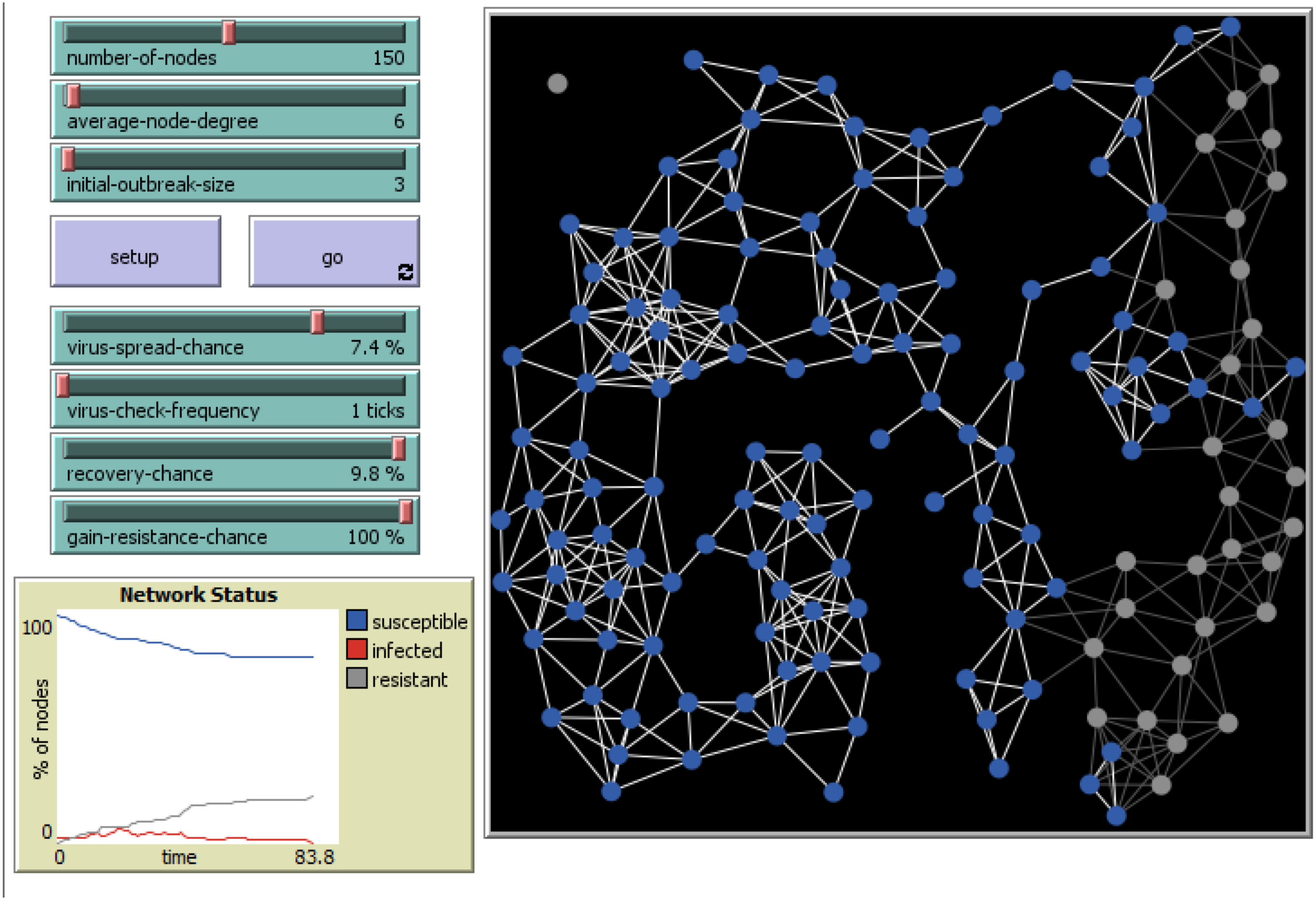
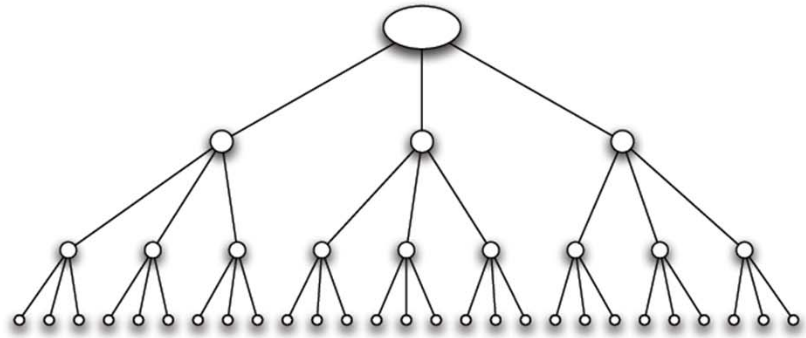
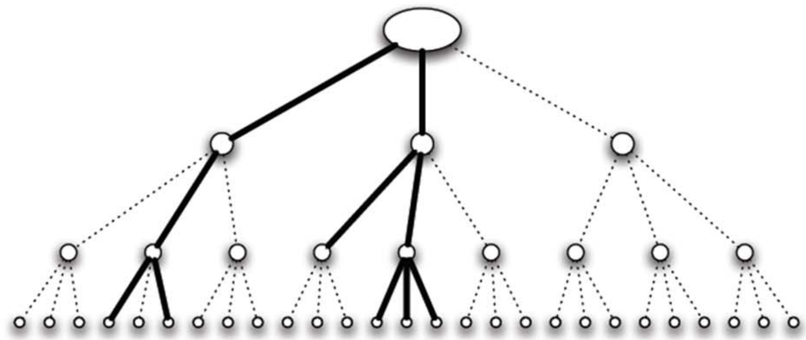



Figure 21.2: The course of an SIR epidemic in which each node remains infectious for a number of steps equal to $t_I = 1$. Starting with nodes $y$ and $z$ initially infected, the epidemic spreads to some but not all of the remaining nodes. In each step, shaded nodes with dark borders are in the Infectious ($I$) state and shaded nodes with thin borders are in the Removed ($R$) state.

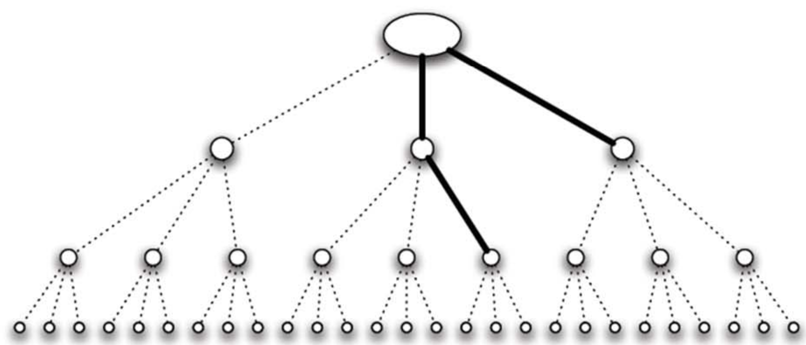

number-of-nodes 150
average-node-degree 6
initial-outbreak-size 3
setup
go
virus-spread-chance 7.4 %
virus-check-frequency 1 ticks
recovery-chance 9.8 %
gain-resistance-chance 100 %
Network Status
100
% of nodes
0
0
time
83.8
susceptible
infected
resistant



(a) *The contact network for a branching process*



(b) *With high contagion probability, the infection spreads widely*



(c) *With low contagion probability, the infection is likely to die out quickly*

Figure 21.1: The branching process model is a simple framework for reasoning about the spread of an epidemic as one varies both the amount of contact among individuals and the level of contagion.

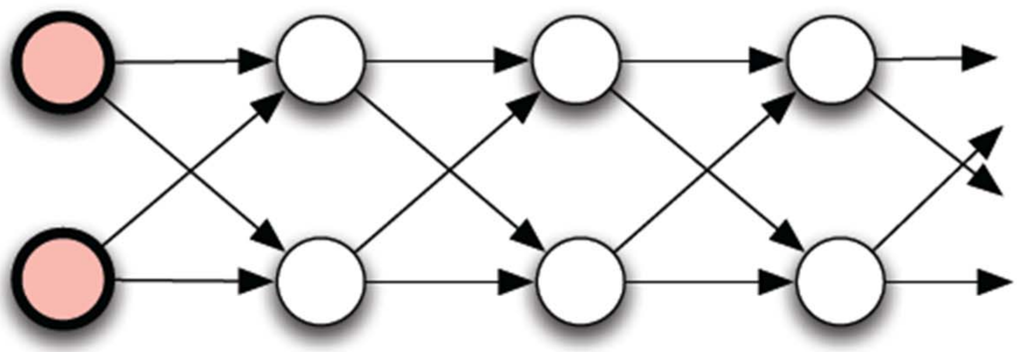
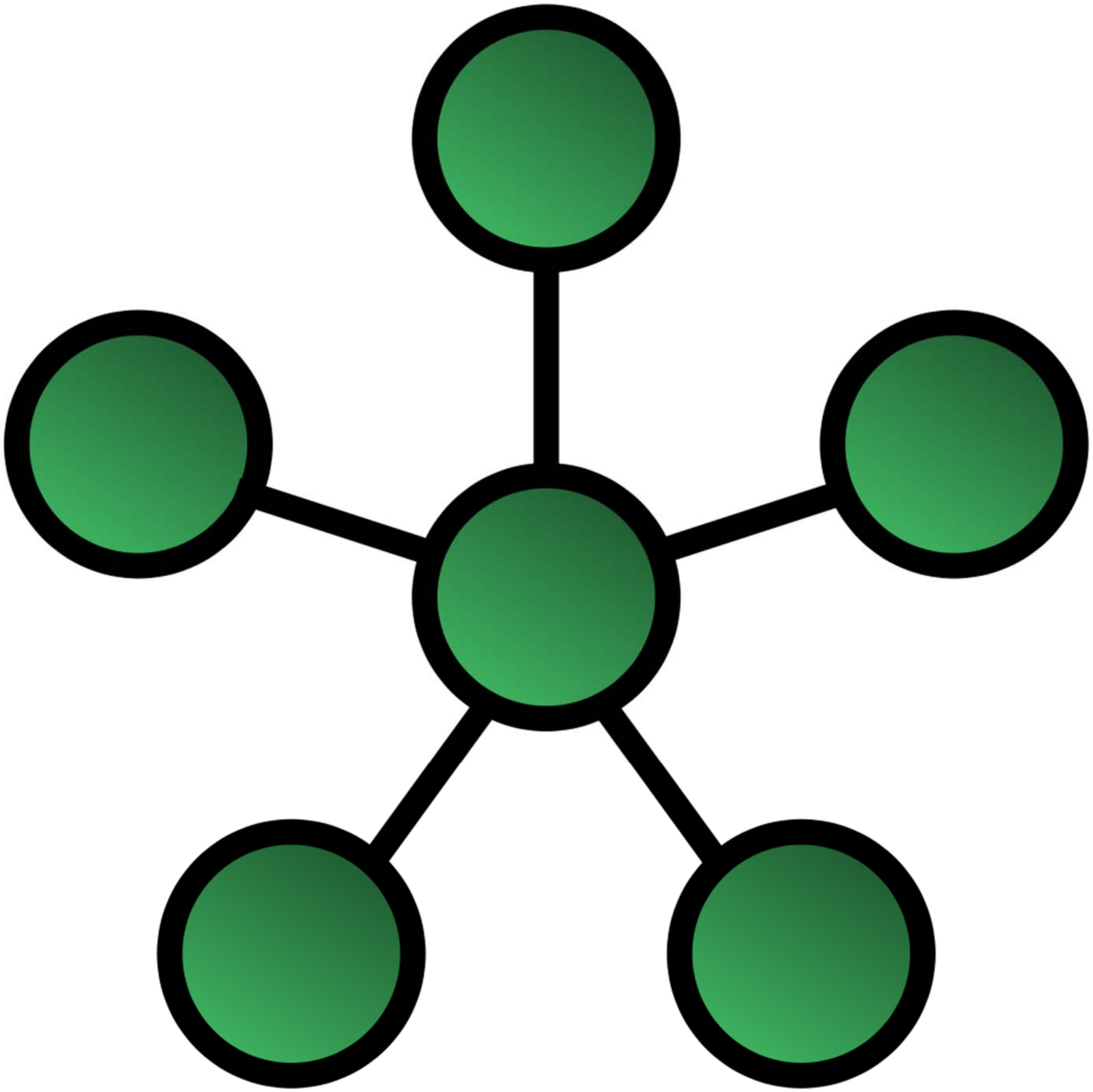


Figure 21.3: In this network, the epidemic is forced to pass through a narrow "channel" of nodes. In such a structure, even a highly contagious disease will tend to die out relatively quickly.



## Basic reproductive number :

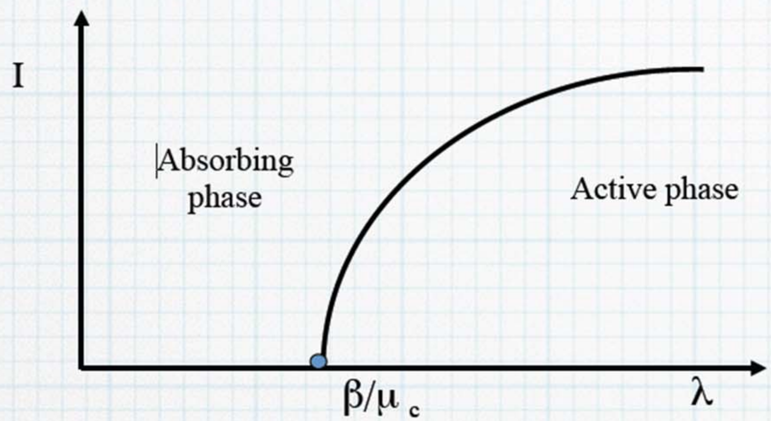
$R_0$ is the average number of individuals infected directly by an infected individuals during his infectious period in a fully susceptible population.

$\beta$ = average number of infections generated in a unit time step

$\tau=1/\mu$ = average time an infectious individual stays infected

$$R_0 = \beta\tau = \frac{\beta}{\mu}$$

# Epidemic threshold

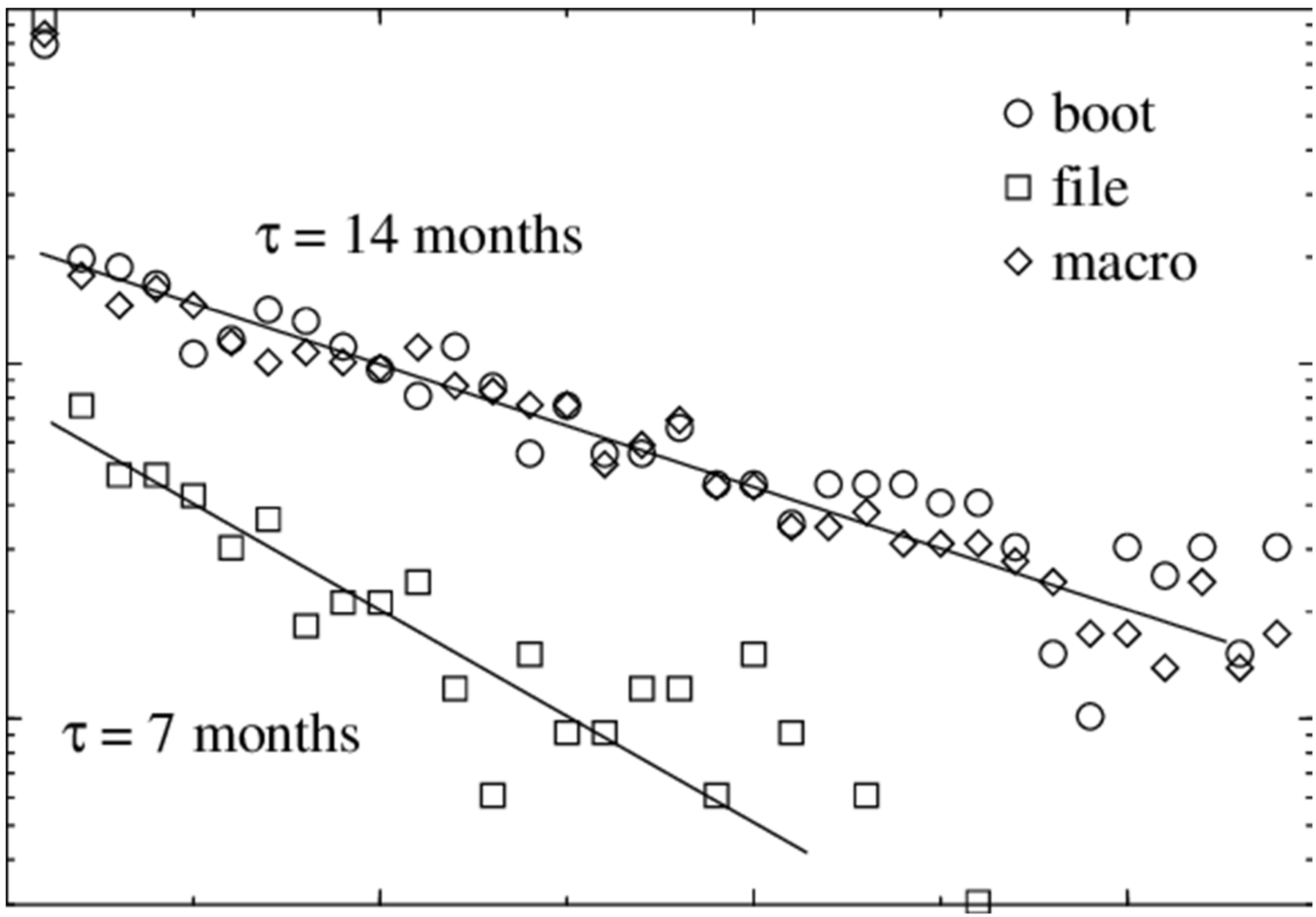


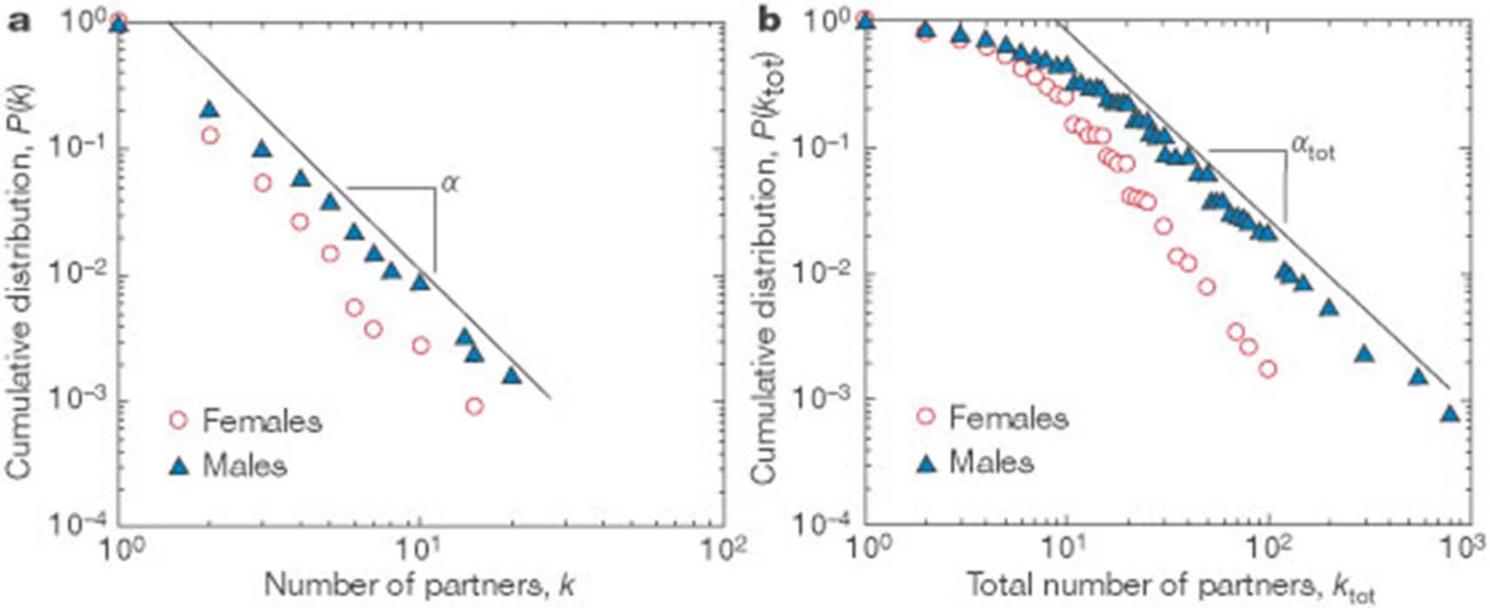

$$\frac{\beta}{\mu} \geq \frac{\langle k \rangle}{\langle k^2 \rangle}$$

**if** $\langle k^2 \rangle \to \infty$ **then** $\beta/\mu \to 0$

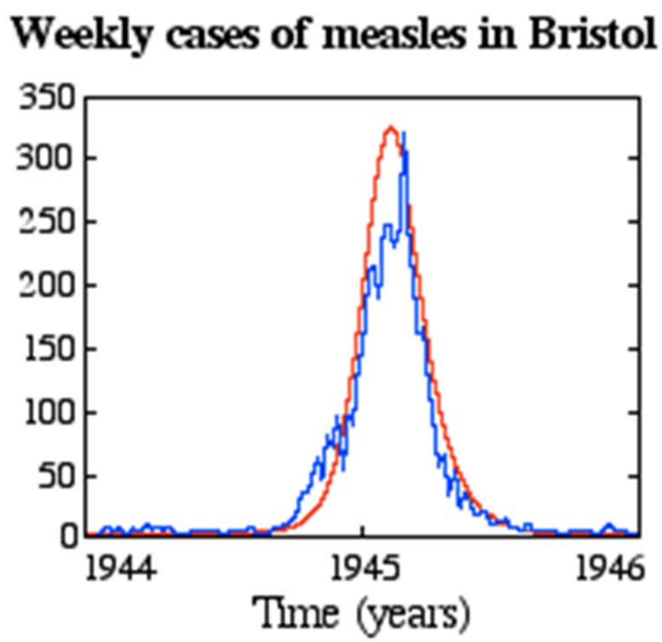
**Heterogeneous networks**

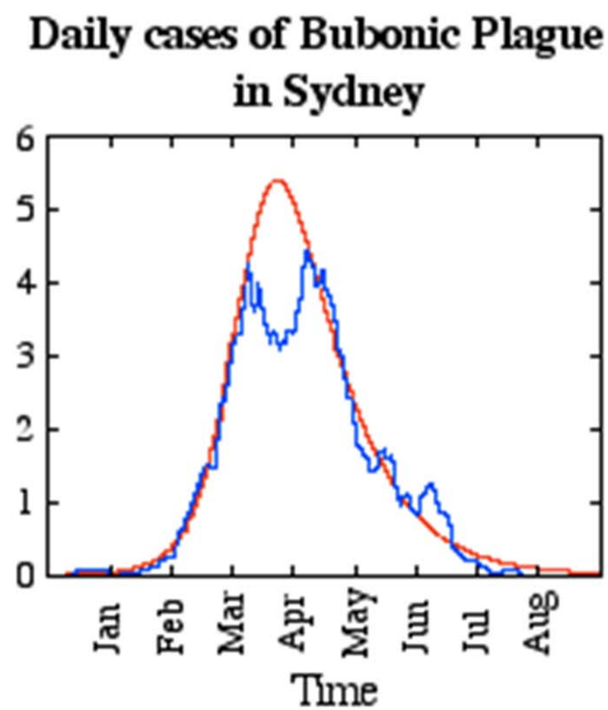
**Vanishing epidemic threshold in the limit of large network with scale-free topology**


boot
file
macro
τ = 14 months
τ = 7 months



a
b
Cumulative distribution, P(k)
Number of partners, k
Females
Males
α
Cumulative distribution, P(k_tot)
Total number of partners, k_tot
α_tot




[www.**youtube**.com/watch?v=UG8YbNbdaco](http://www.youtube.com/watch?v=UG8YbNbdaco)

$$\frac{dS}{dt} = -\beta SI$$

$$\frac{dI}{dt} = \beta SI - \mu I$$

$$\frac{dR}{dt} = \mu I$$

1. Equilibrium points
2. Beta < Mu
3. Mu > Beta
4. R lim_t-> Inf
5. R_0
6. Extension to networks

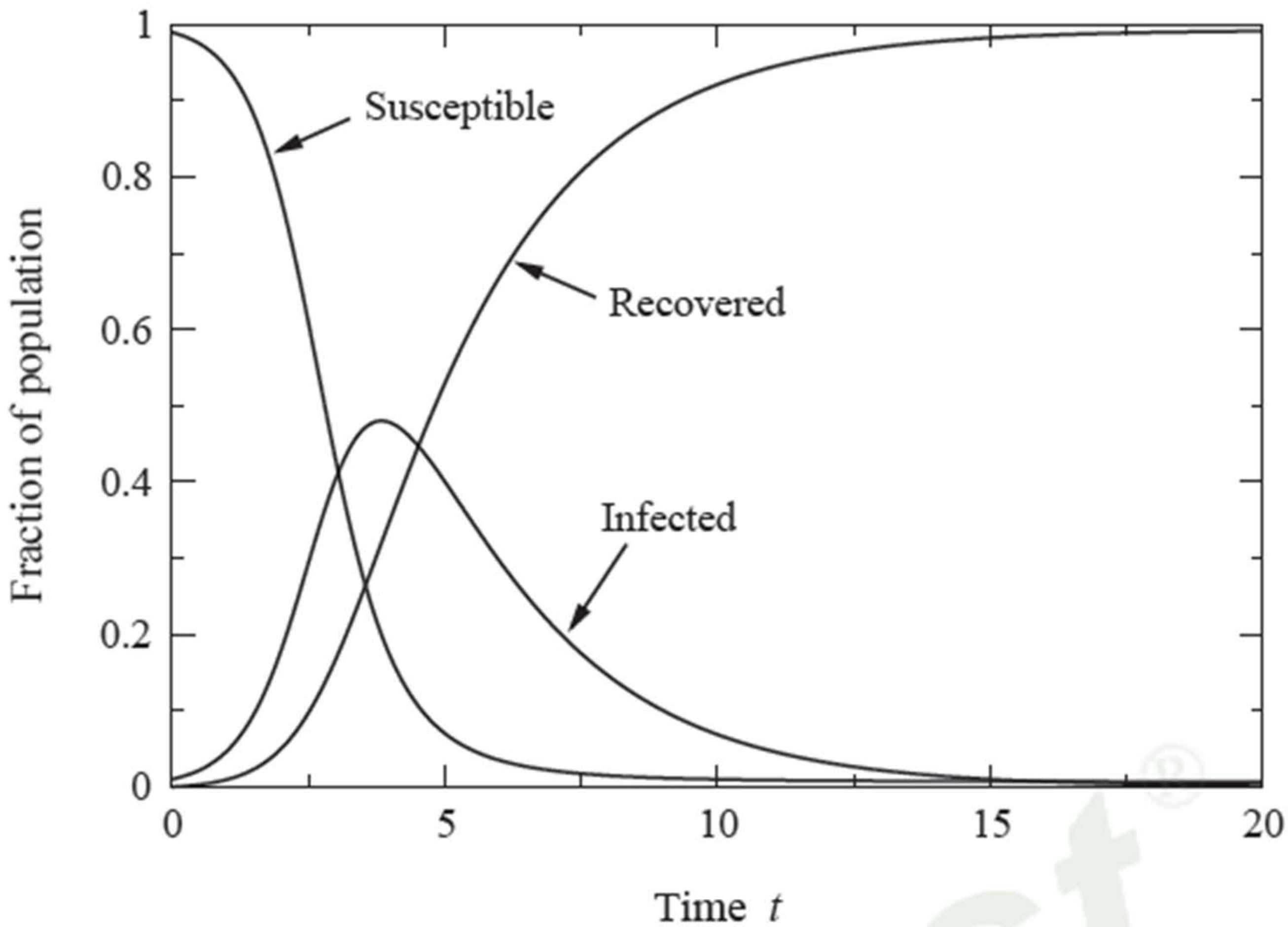
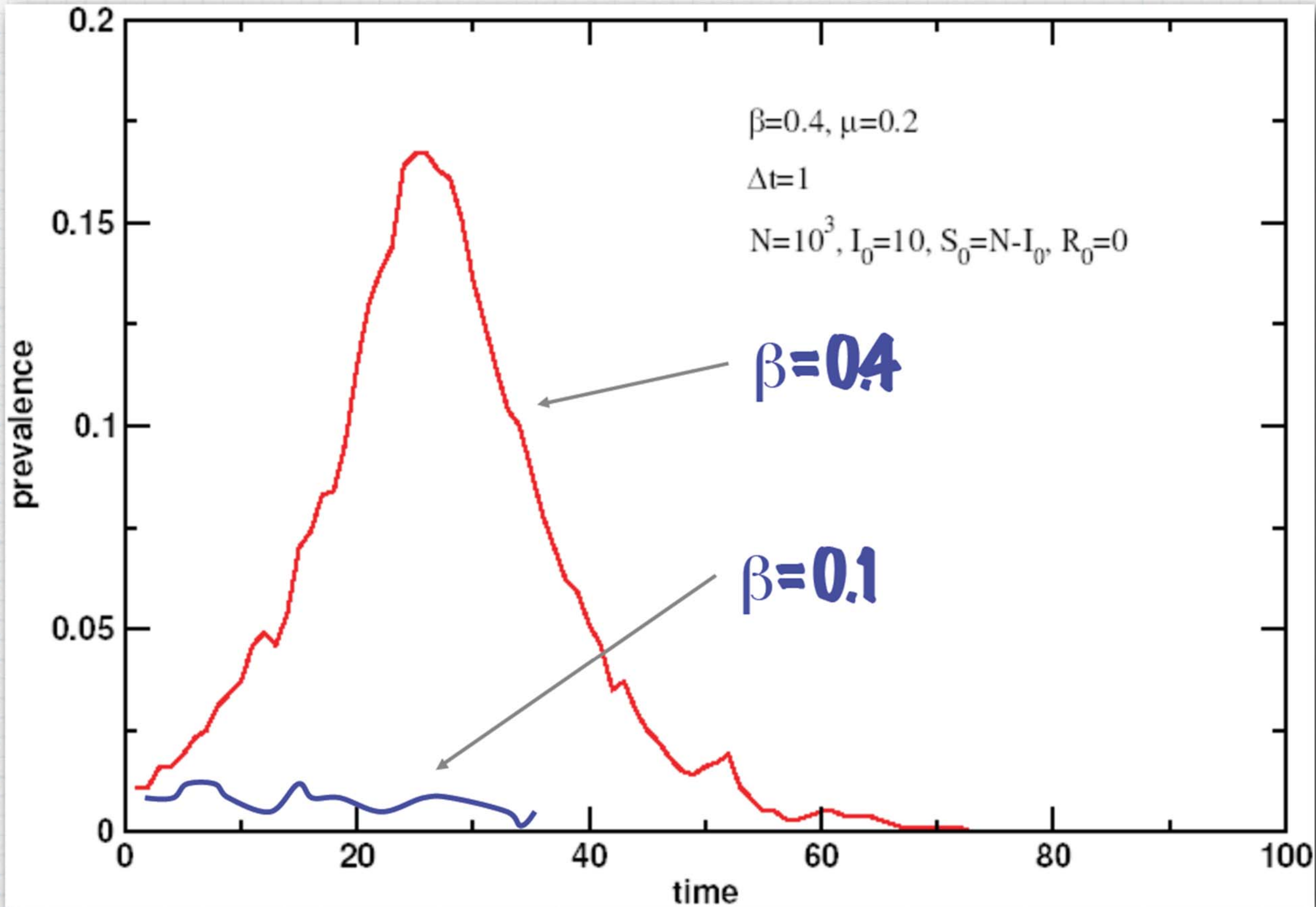
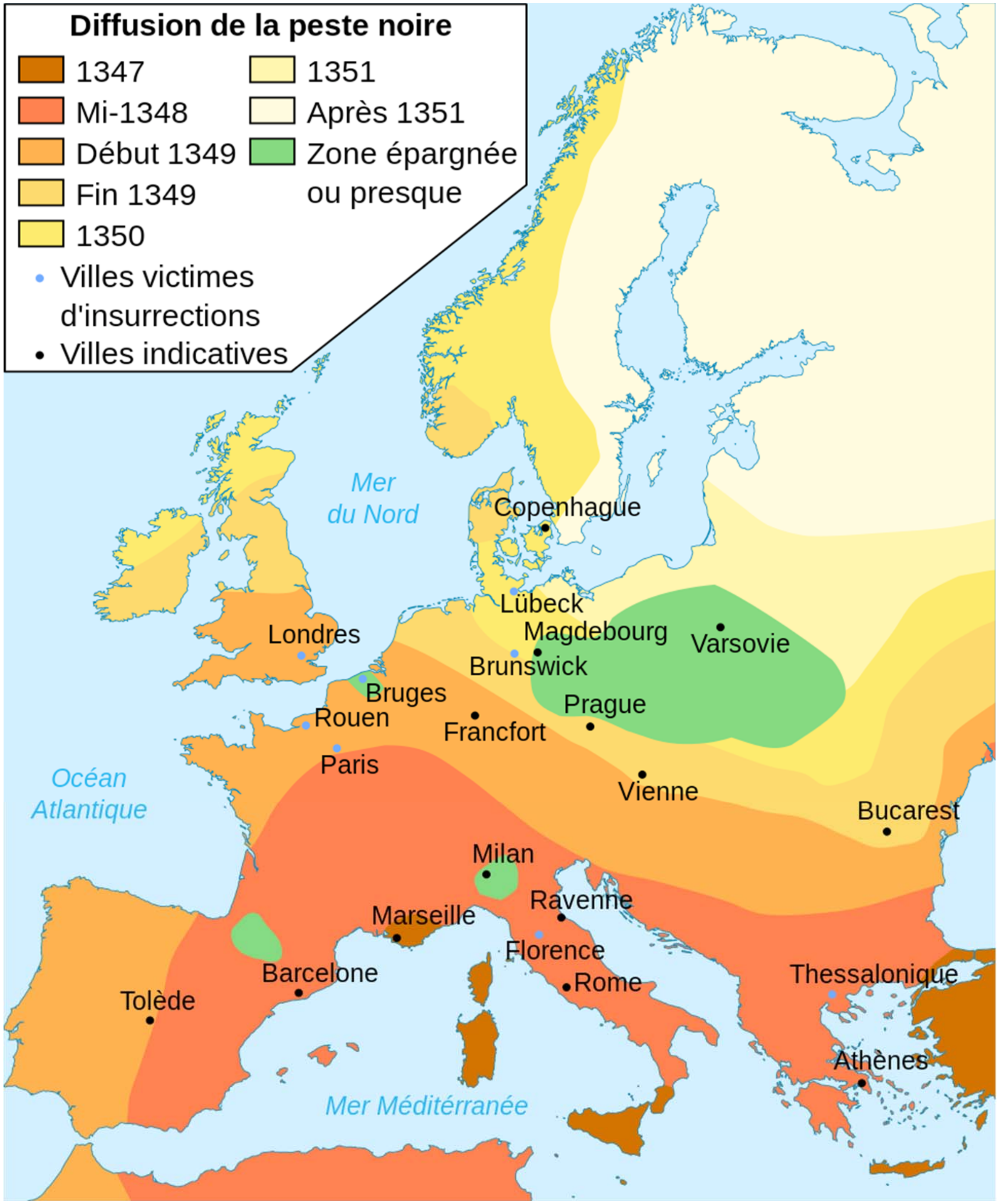
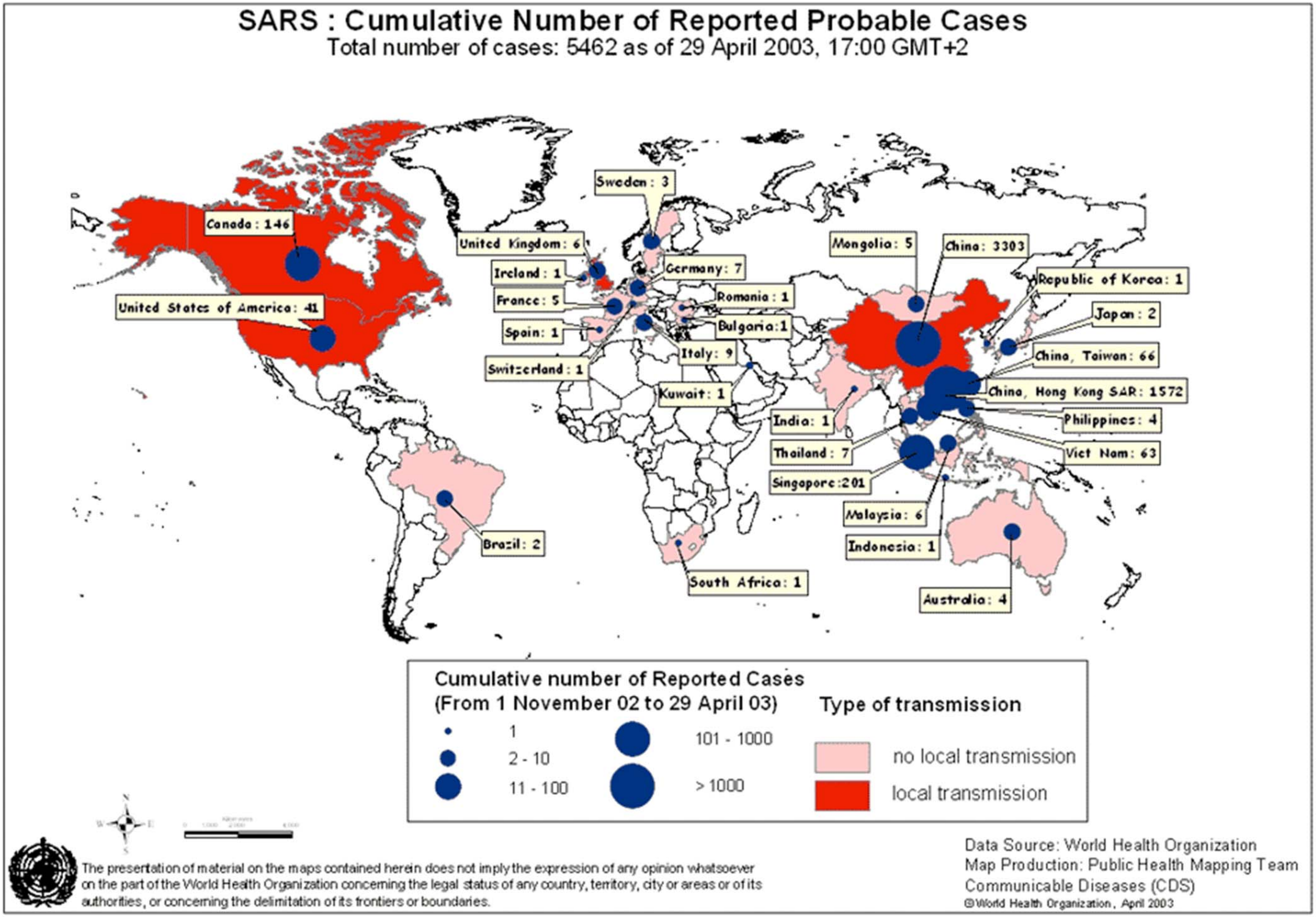


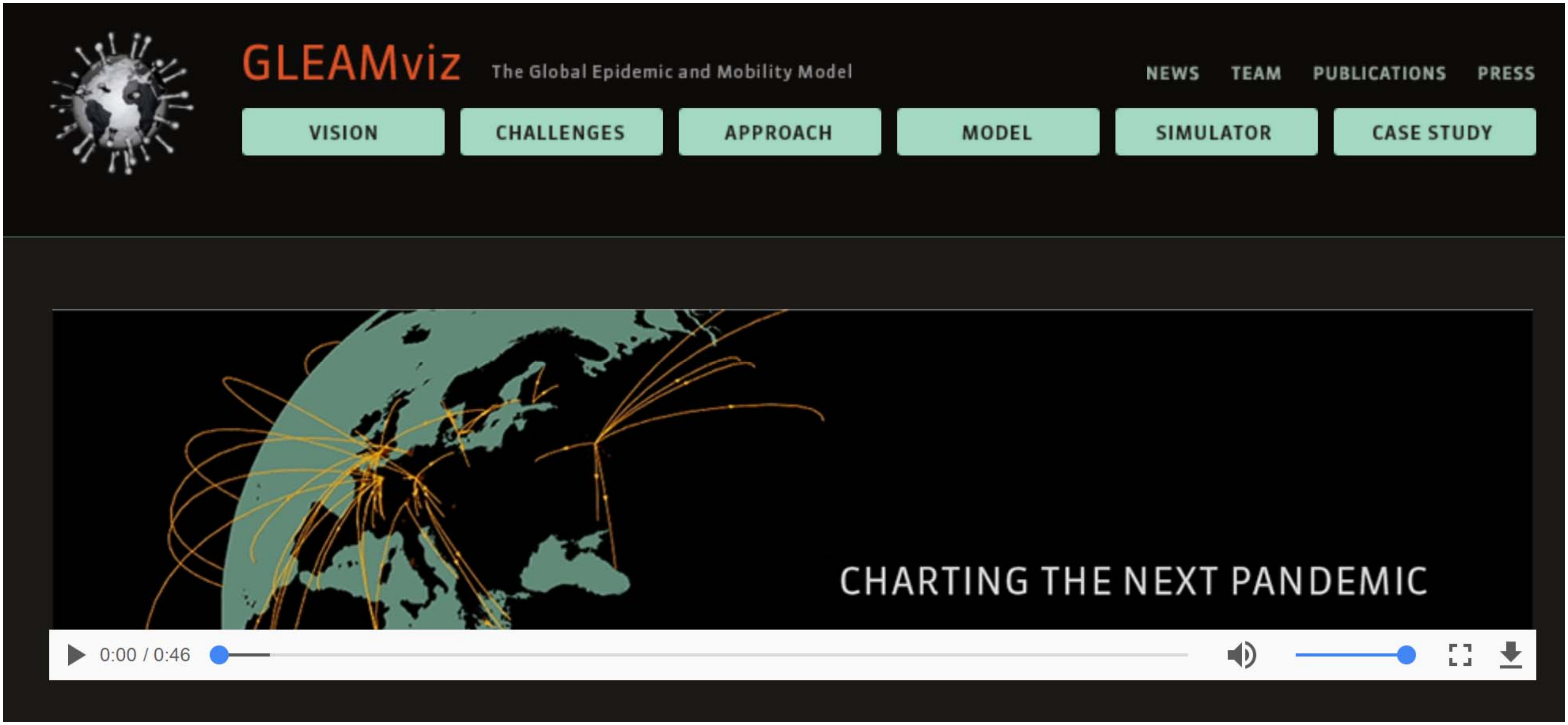

**Figure 17.2: Time evolution of the SIR model.** The three curves in this figure show the fractions of the population in the susceptible, infected, and recovered states as a function of time. The parameters are $\beta = 1$, $\gamma = 0.4$, $s_0 = 0.99$, $x_0 = 0.01$, and $r_0 = 0$.

# SIR model


β=0.4, μ=0.2
Δt=1
N=10^3, I_0=10, S_0=N-I_0, R_0=0
β=0.4
β=0.1
prevalence
time
0
0.05
0.1
0.15
0.2
20
40
60
80
100


Diffusion de la peste noire
1347
Mi-1348
Début 1349
Fin 1349
1350
1351
Après 1351
Zone épargnée ou presque
Villes victimes d'insurrections
Villes indicatives
Mer du Nord
Océan Atlantique
Mer Méditérranée
Copenhague
Lübeck
Magdebourg
Varsovie
Brunswick
Londres
Bruges
Rouen
Francfort
Prague
Paris
Vienne
Bucarest
Milan
Ravenne
Marseille
Florence
Barcelone
Rome
Tolède
Thessalonique
Athènes


SARS : Cumulative Number of Reported Probable Cases
Total number of cases: 5462 as of 29 April 2003, 17:00 GMT+2
Canada: 146
United States of America: 41
United Kingdom: 6
Ireland: 1
France: 5
Spain: 1
Switzerland: 1
Sweden: 3
Germany: 7
Romania: 1
Bulgaria: 1
Italy: 9
Kuwait: 1
Mongolia: 5
China: 3303
Republic of Korea: 1
Japan: 2
China, Taiwan: 66
China, Hong Kong SAR: 1572
Philippines: 4
Viet Nam: 63
India: 1
Thailand: 7
Singapore: 201
Malaysia: 6
Indonesia: 1
Brazil: 2
South Africa: 1
Australia: 4
Cumulative number of Reported Cases
(From 1 November 02 to 29 April 03)
1
2 - 10
11 - 100
101 - 1000
> 1000
Type of transmission
no local transmission
local transmission
The presentation of material on the maps contained herein does not imply the expression of any opinion whatsoever on the part of the World Health Organization concerning the legal status of any country, territory, city or areas or of its authorities, or concerning the delimitation of its frontiers or boundaries.
Data Source: World Health Organization
Map Production: Public Health Mapping Team
Communicable Diseases (CDS)
©World Health Organization, April 2003


GLEAMviz
The Global Epidemic and Mobility Model
NEWS
TEAM
PUBLICATIONS
PRESS
VISION
CHALLENGES
APPROACH
MODEL
SIMULATOR
CASE STUDY
CHARTING THE NEXT PANDEMIC
0:00 / 0:46

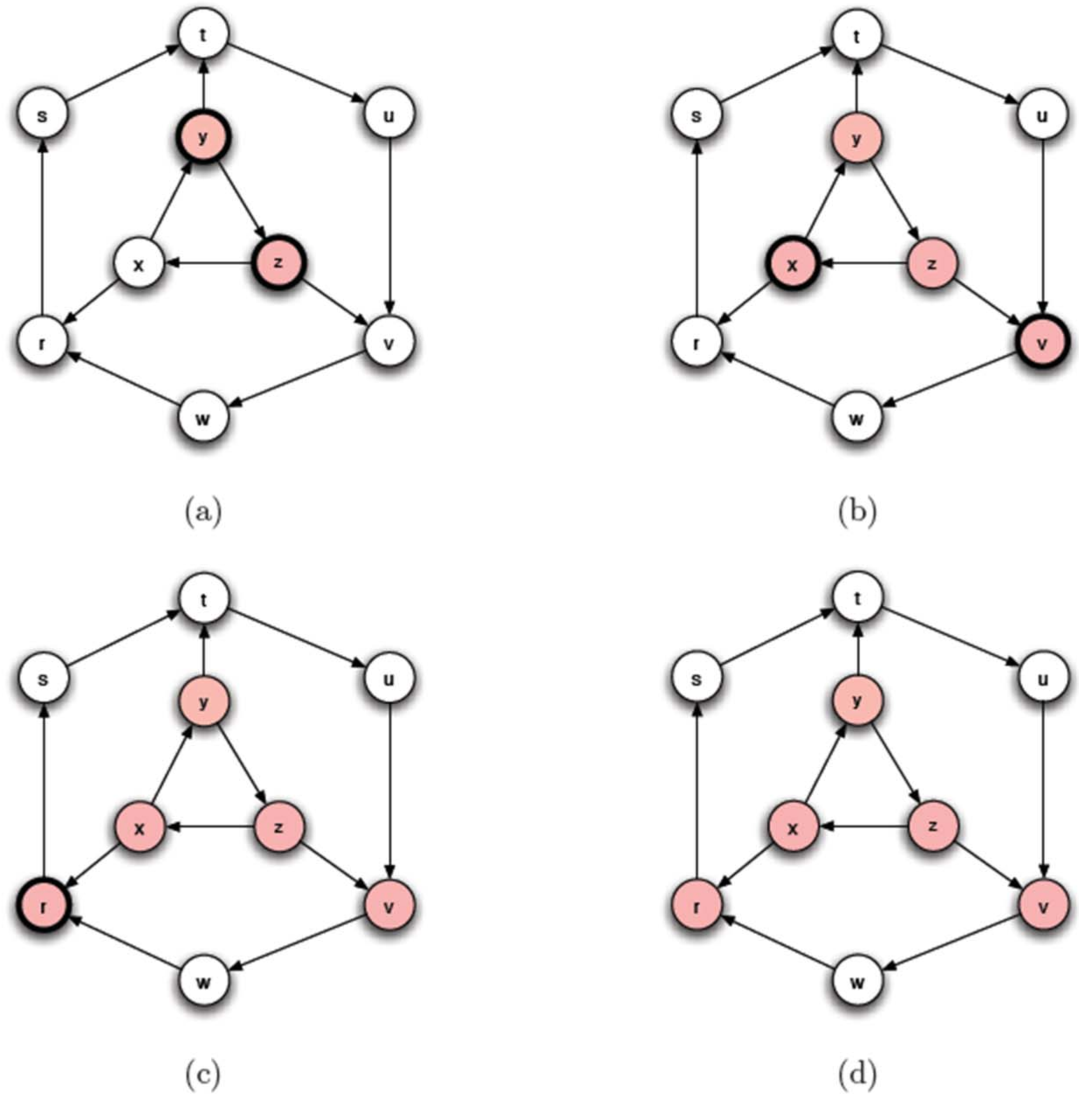



Figure 21.2: The course of an SIR epidemic in which each node remains infectious for a number of steps equal to $t_I = 1$. Starting with nodes $y$ and $z$ initially infected, the epidemic spreads to some but not all of the remaining nodes. In each step, shaded nodes with dark borders are in the Infectious ($I$) state and shaded nodes with thin borders are in the Removed ($R$) state.

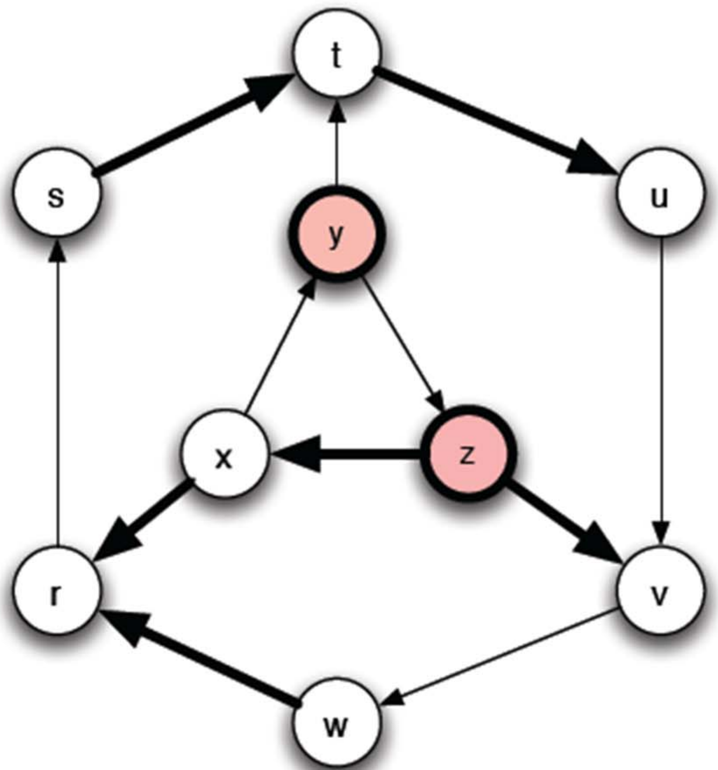
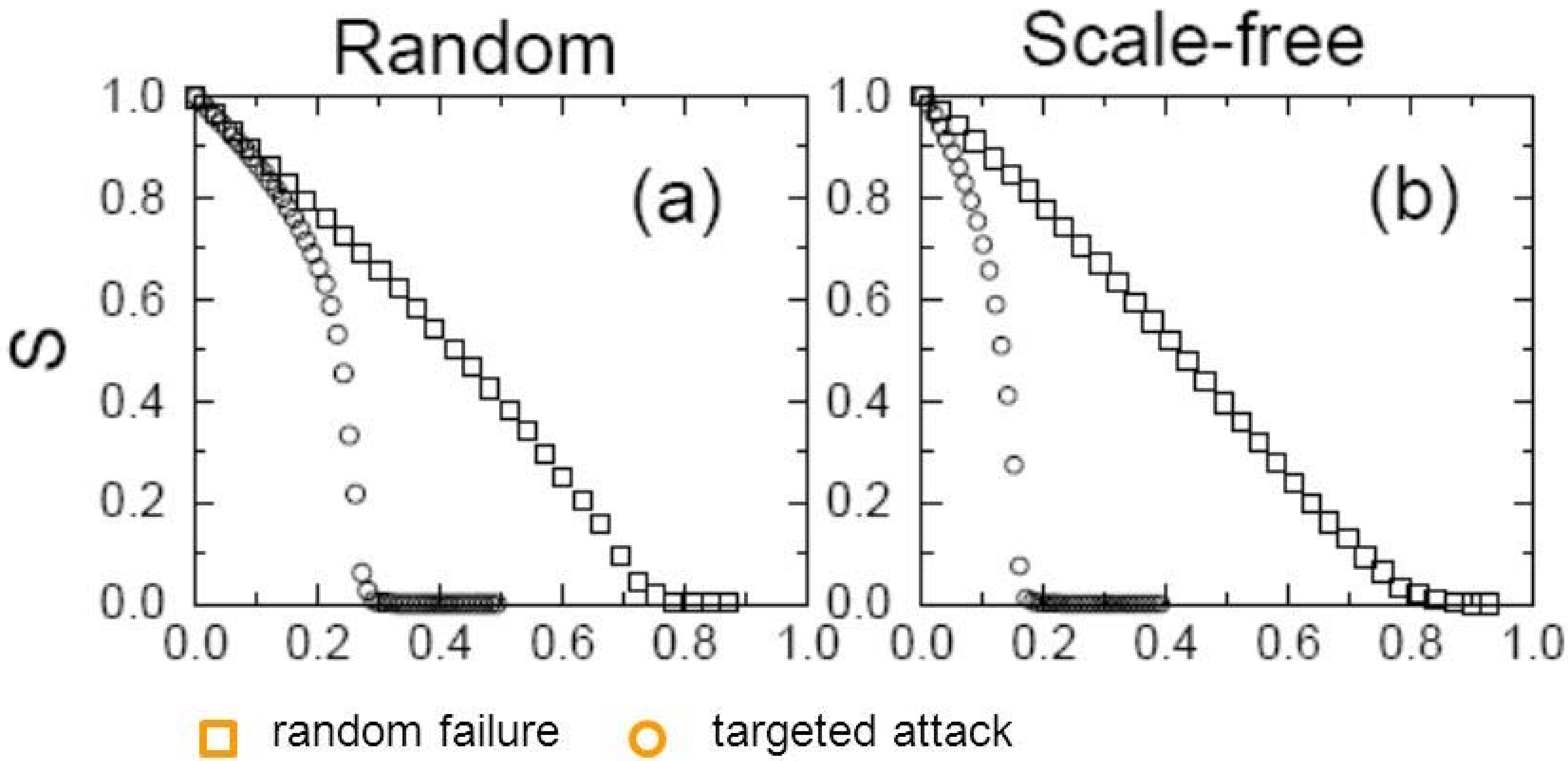


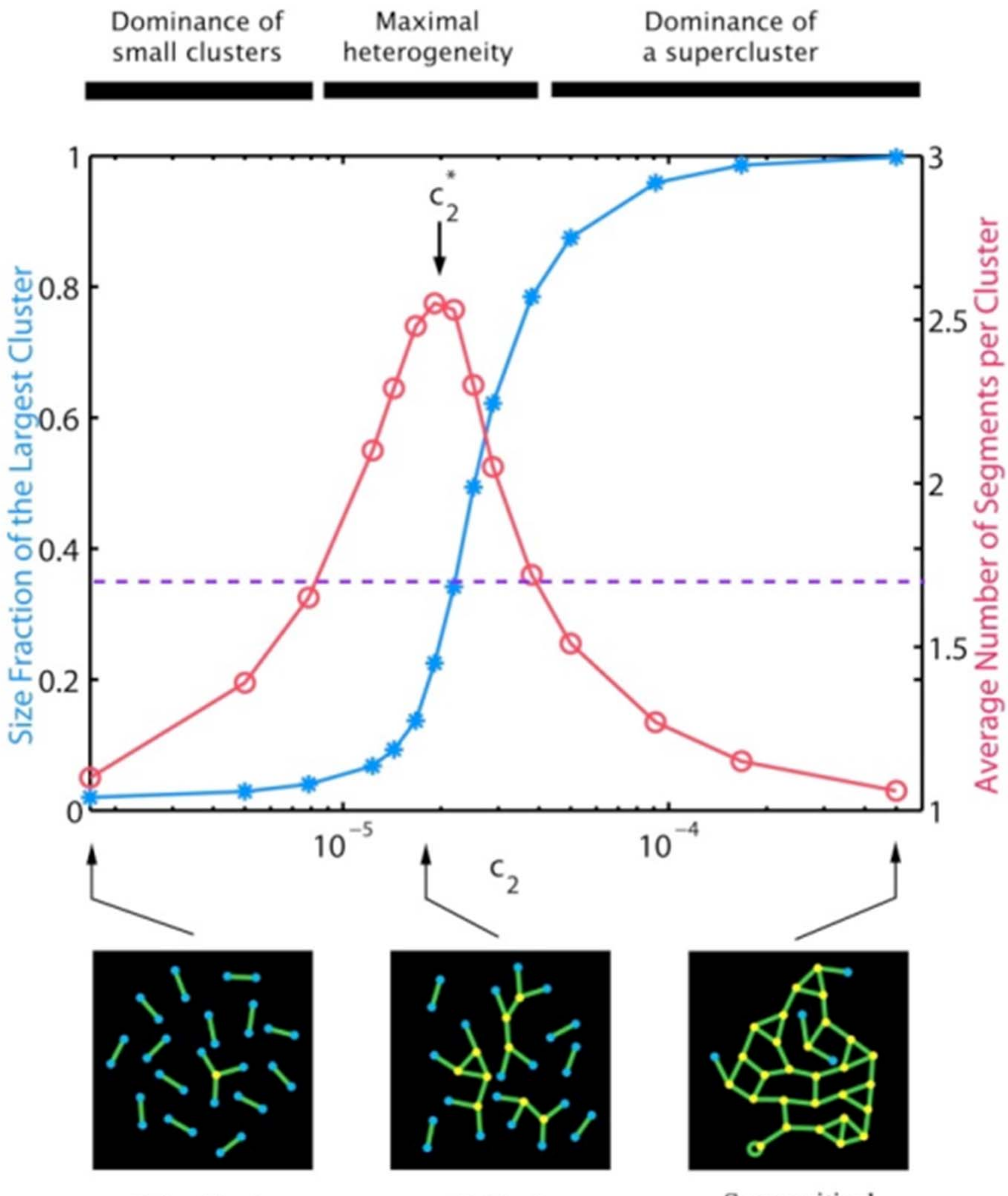

Figure 21.4: An equivalent way to view an SIR epidemic is in terms of *percolation*, where we decide in advance which edges will transmit infection (should the opportunity arise) and which will not.

# Network resilience to targeted attacks

- Scale-free graphs are resilient to random attacks, but sensitive to targeted attacks.
- For random networks there is smaller difference between the two



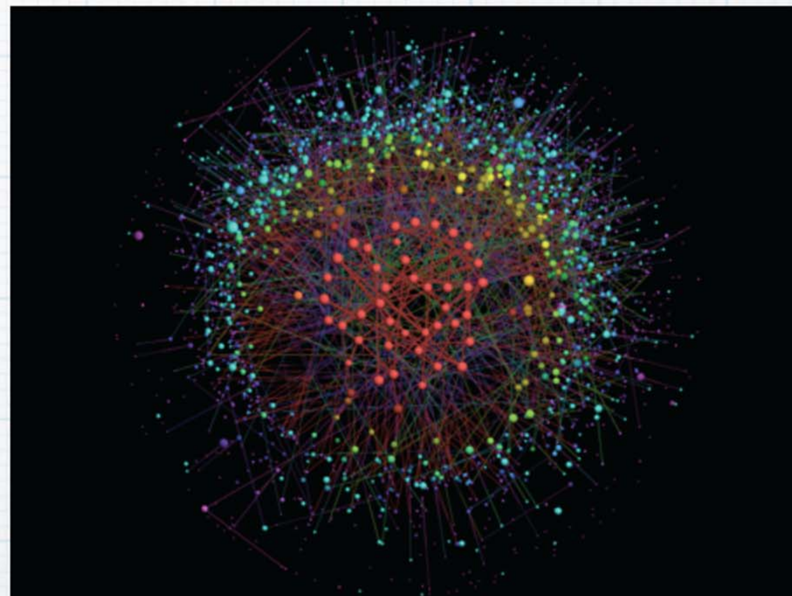

Source: Error and attack tolerance of complex networks. Réka Albert, Hawoong Jeong and Albert-László Barabási



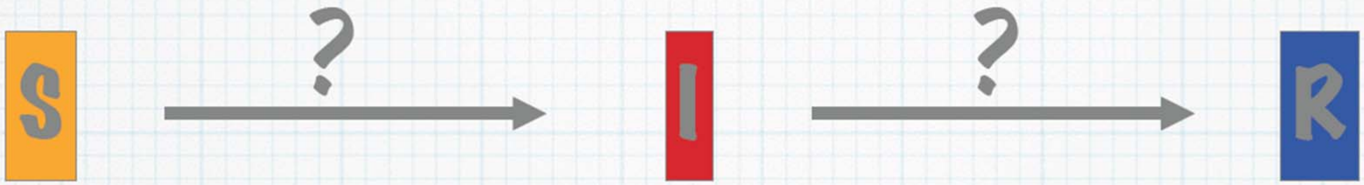
Dominance of small clusters
Maximal heterogeneity
Dominance of a supercluster
Size Fraction of the Largest Cluster
Average Number of Segments per Cluster
$c_2^*$
$c_2$
$10^{-5}$
$10^{-4}$
0
0.2
0.4
0.6
0.8
1
1.5
2
2.5
3
Subcritical
Critical
Supercritical

# Diffusion and epidemic processes in real-world networks

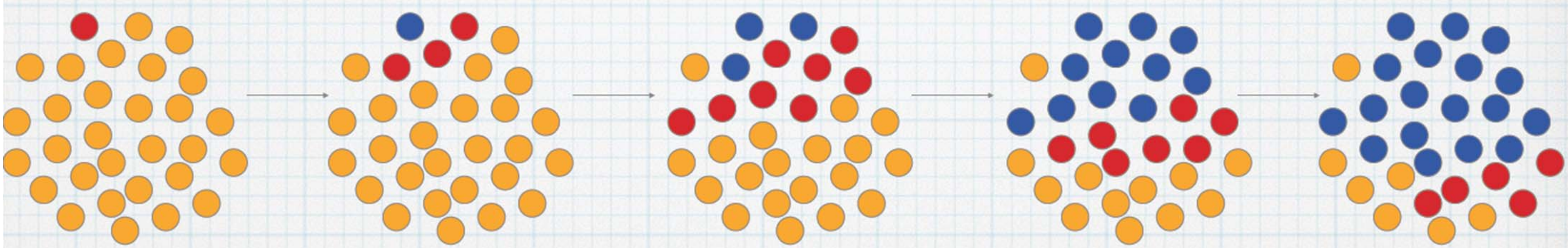
- Epidemic processes
- Opinion spreading
- Stochastic data dissemination
- Packets routing
- Pattern formation
- Transport in random media
- ...



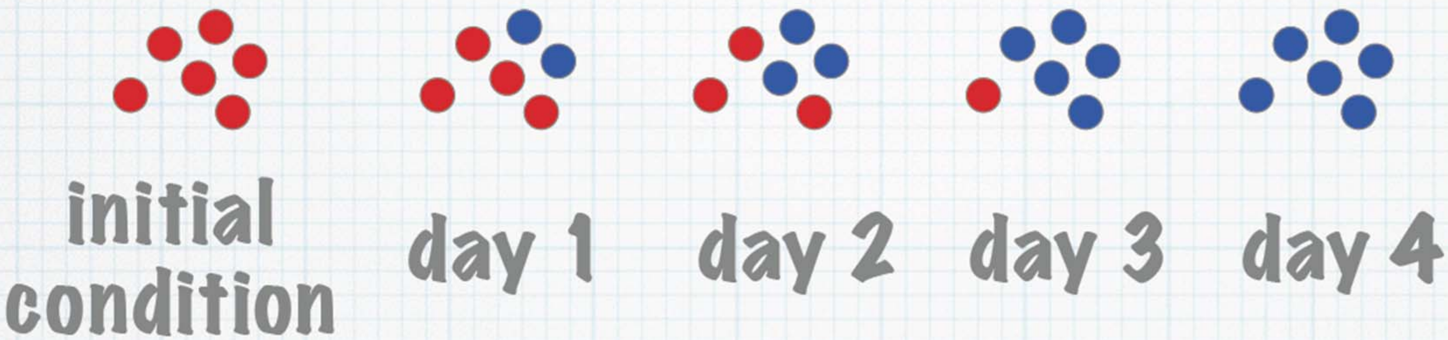
# SIR model




dynamics ?



# SIR model: I → R



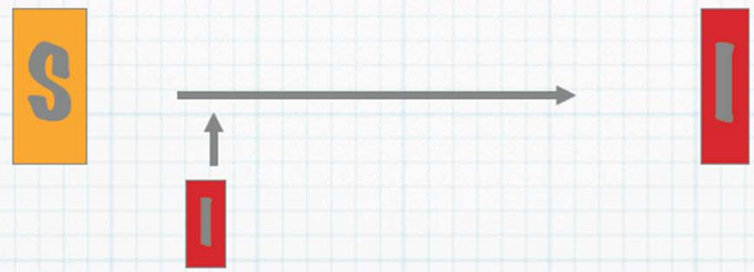

$$\mu = \frac{1}{\tau}$$

rate of recovery = inverse of average infectious period

$$\Delta I = \mu I \Delta t$$

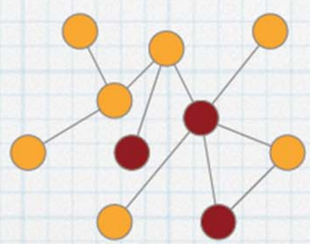
average number of infected recovering in the time interval $\Delta t$

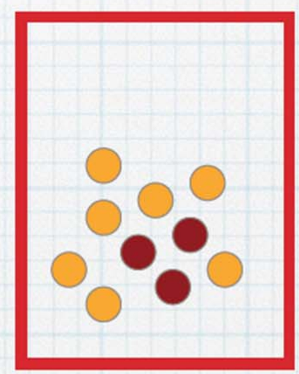
# compartments: 2-body interactions



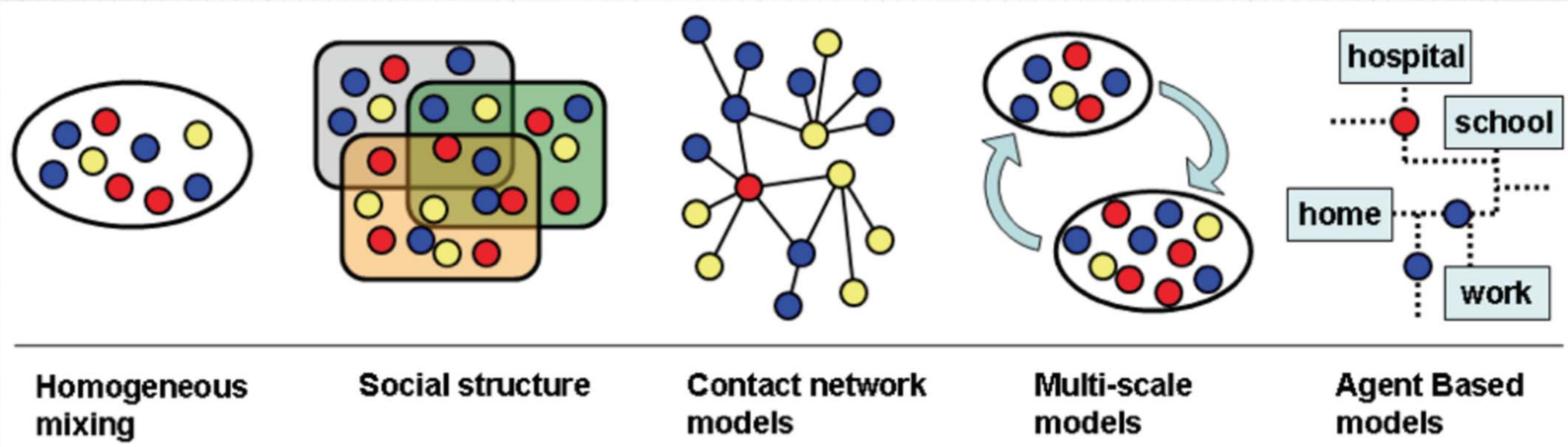

$$\Delta S = \beta \times S \frac{I}{N} \Delta t$$

RANDOM MIXING !





# Wide spectrum of complications and complex features to include...




**Simple** → **Realistic**

Ability to explain (caveats) trends at a population level

Model realism at cost of transparency.
Validation is harder

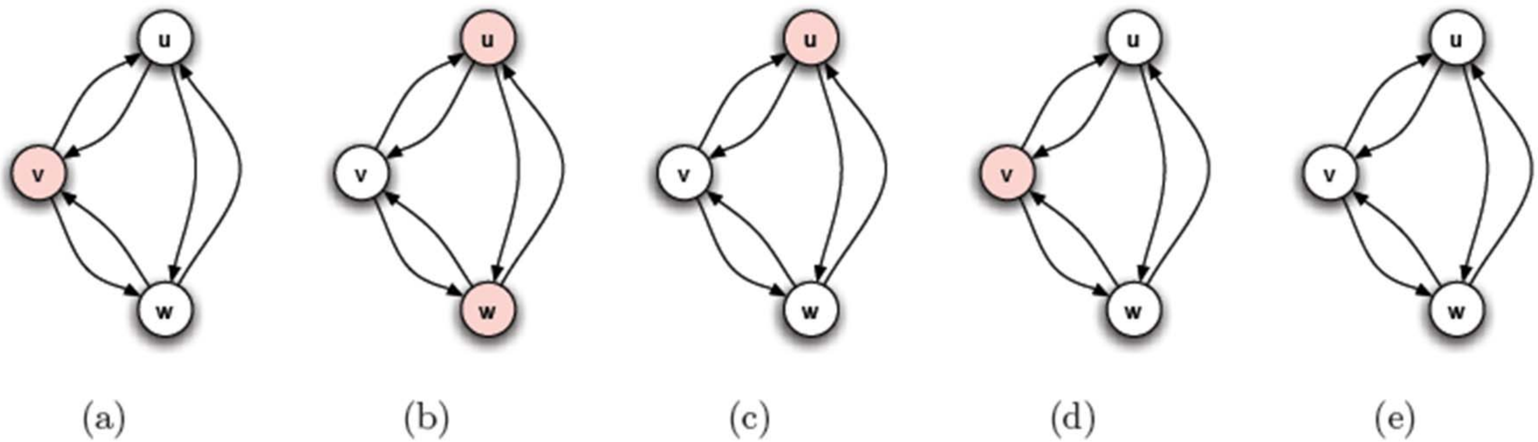
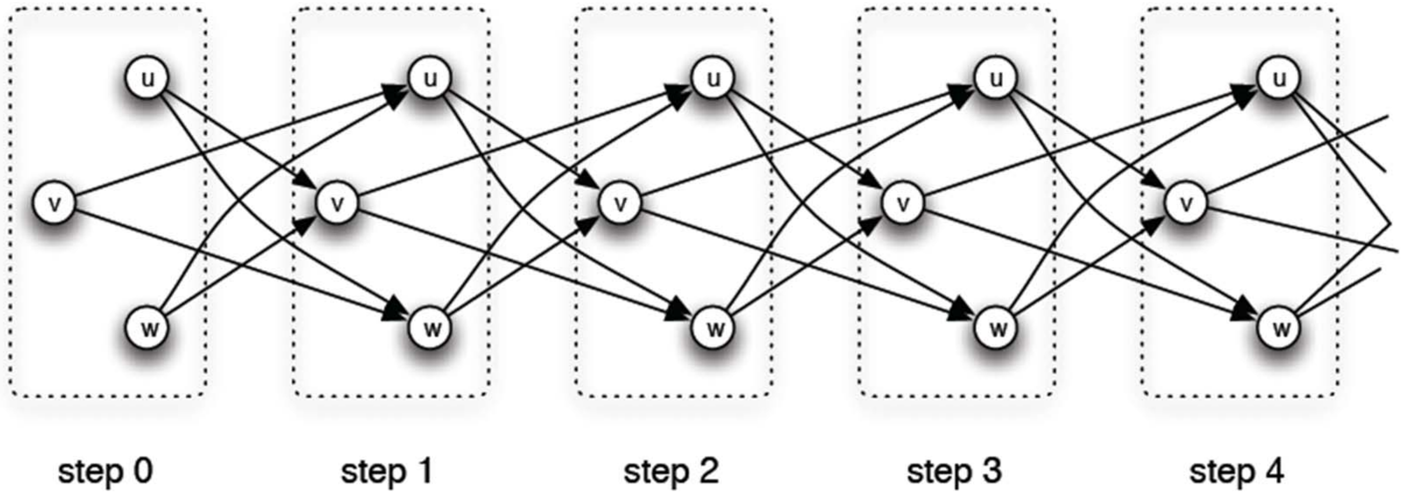
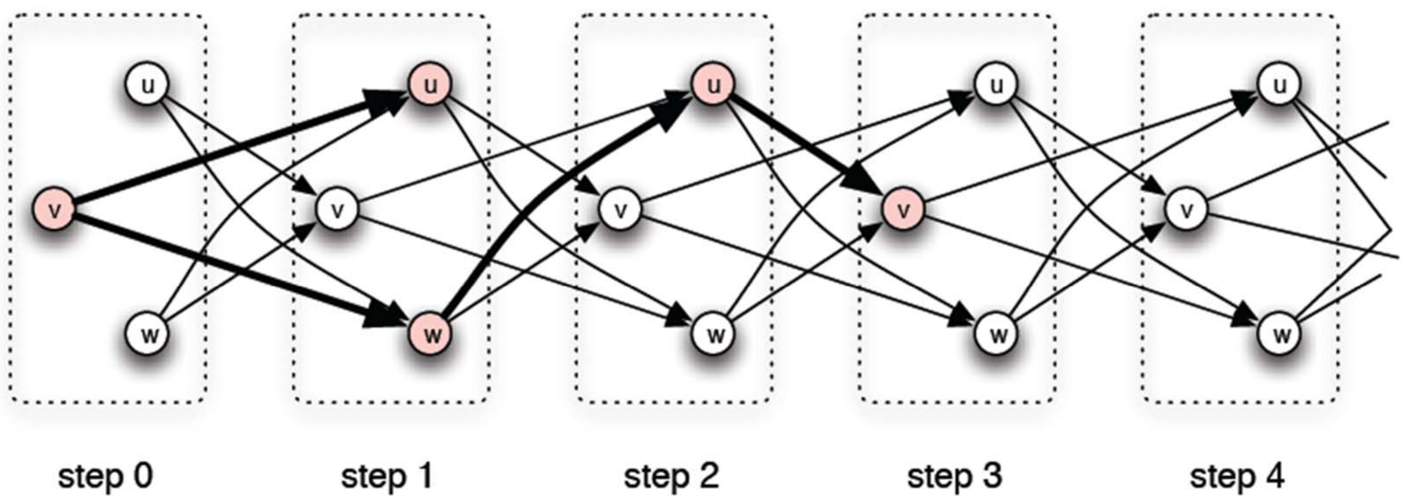



Figure 21.5: In an SIS epidemic, nodes can be infected, recover, and then be infected again. In each step, the nodes in the Infectious state are shaded.




(a) *To represent the SIS epidemic using the SIR model, we use a "‘time-expanded" contact network*




(b) *The SIS epidemic can then be represented as an SIR epidemic on this time-expanded network.*

Figure 21.6: An SIS epidemic can be represented in the SIR model by creating a separate copy of the contact network for each time step: a node at time $t$ can infect its contact neighbors at time $t+1$.