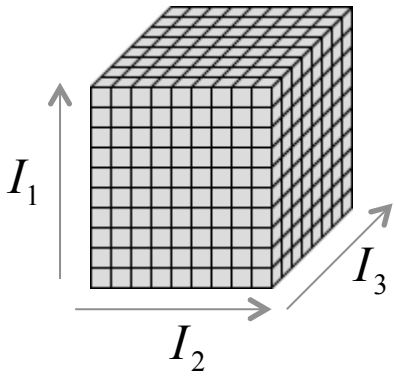
Tensor Representation

- Tensors are multidimensional arrays (generalization of matrices)
- Used to represent data with a number of features greater that two
  - Documents – Keywords – Years
  - Author – Keyword – Venue
  - Source – Destination – Time



Introduction

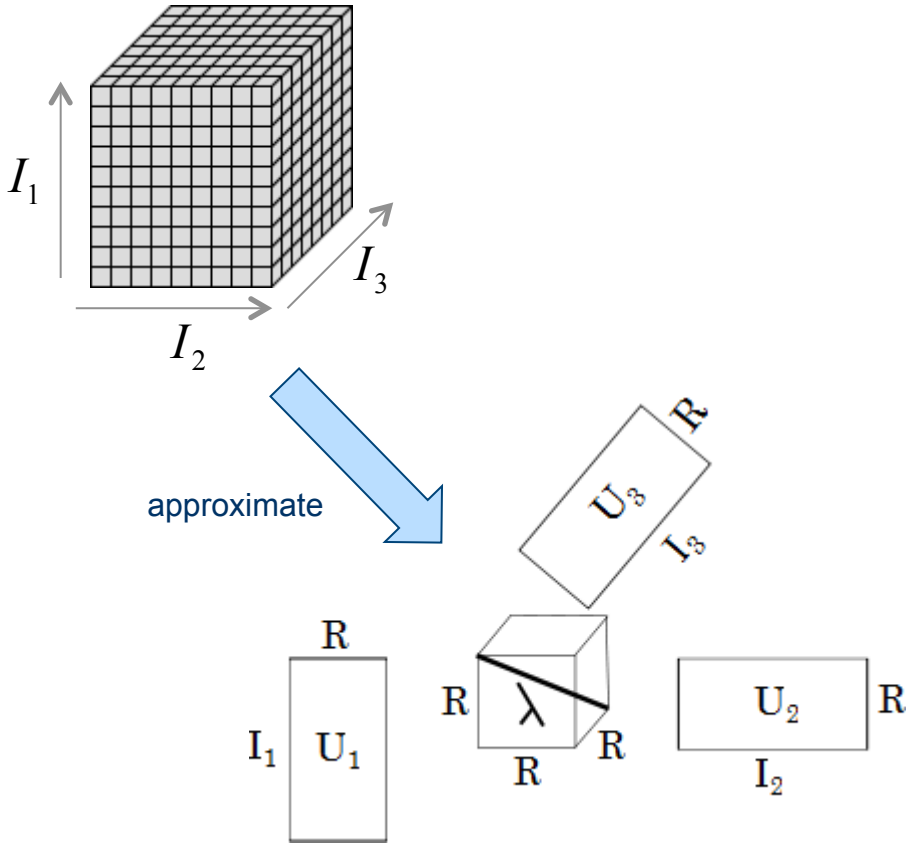
- Tensors have been widely used in many areas
  - Psychometrics [Tucker, 1966]
  - Information retrieval [Chew et al., 2007]
  - Sensor networks analysis [Sun et al., 2007]
  - Web ranking and analysis [Kolda et al, 2006; Sun et al., 2005]

Tensor decompositions

- Multi-way data analysis
- They allow the extraction of
  - hidden correlations among data
  - clusters of data
  - degree of contribution of each data element to relationships

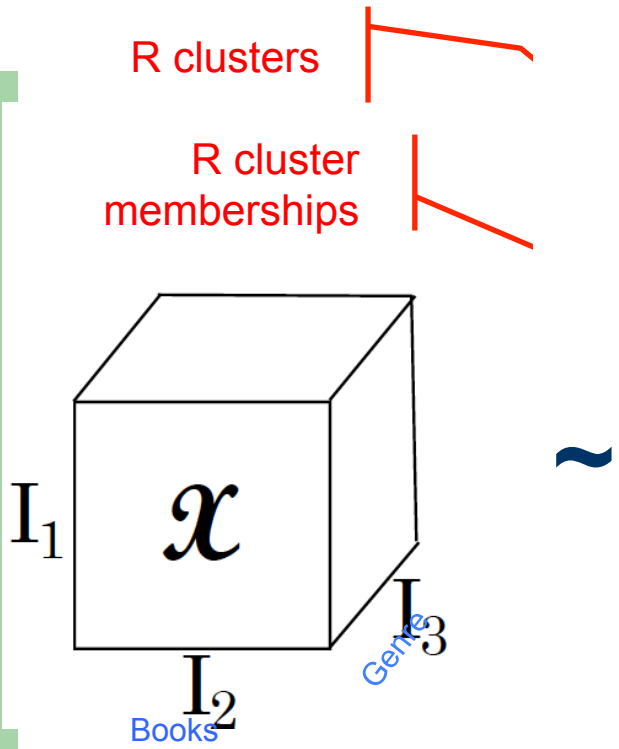
Tensor decompositions

- CANDECOMP / PARAllel FACtors (PARAFAC) [Carrol et al., 1970; Harshman, 1970]
  - Sum of R rank-one tensors
  - Diagonal tensor
  - Set of factor matrices



Tensor Representation of the Data

- Tensor decomposition [CP,Tucker] can be used for
  - understanding spectral characteristics of the data and
  - clustering the data based on inter-dependencies.



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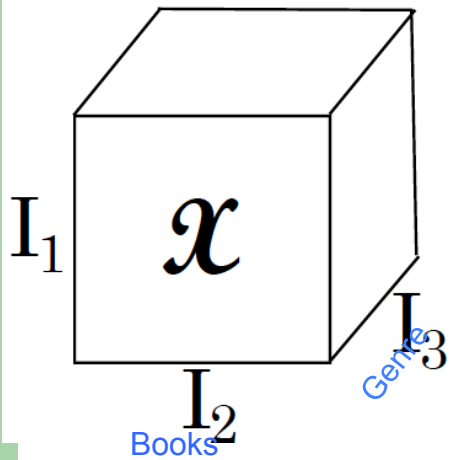
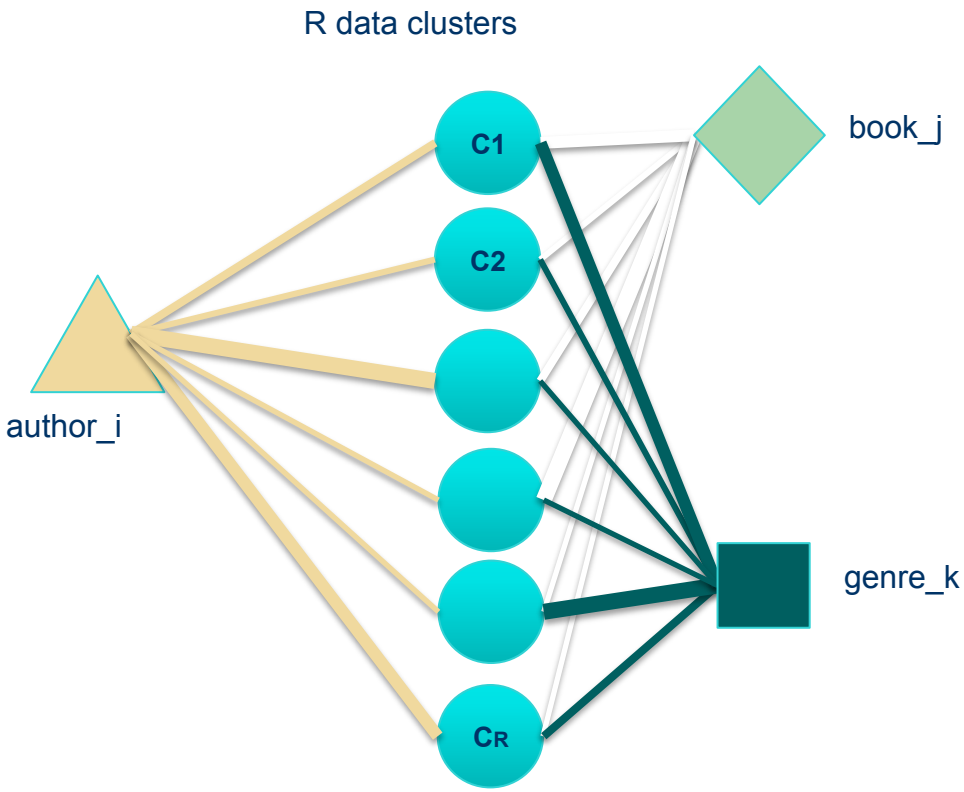
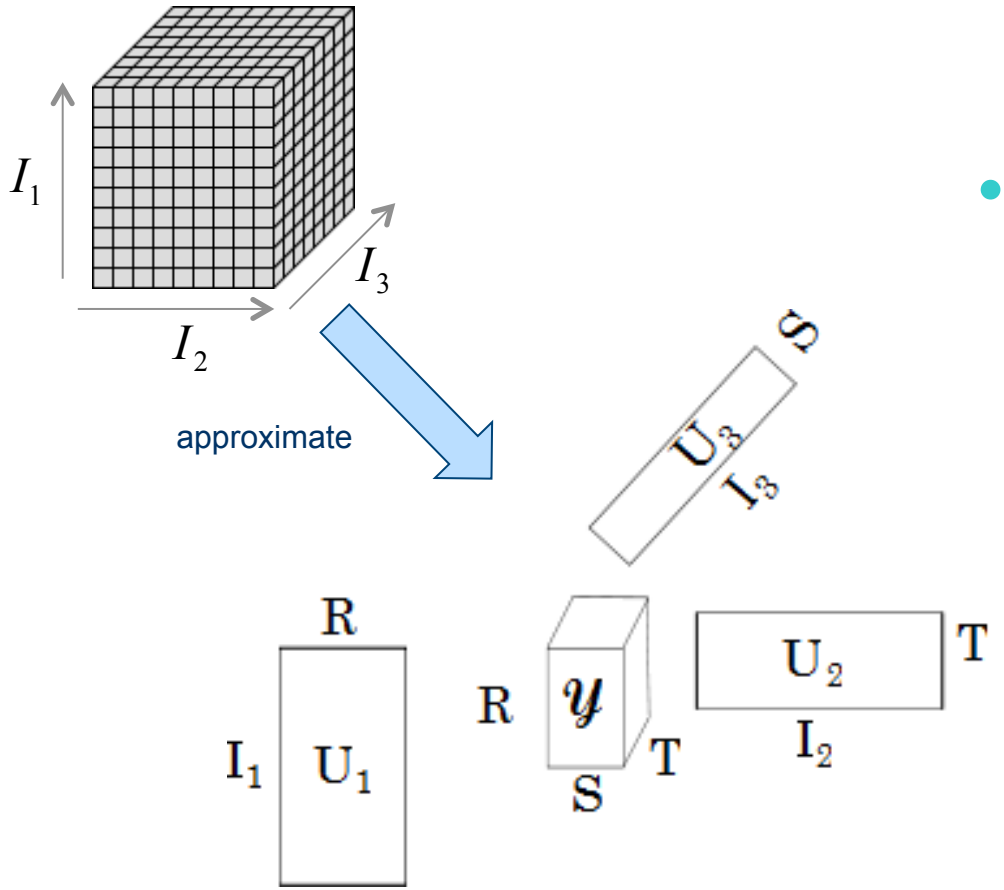




Tensor decompositions



- Tucker [Tucker, 1966]
  - Dense core tensor
  - Set of matrices that represent subspaces of dimensions

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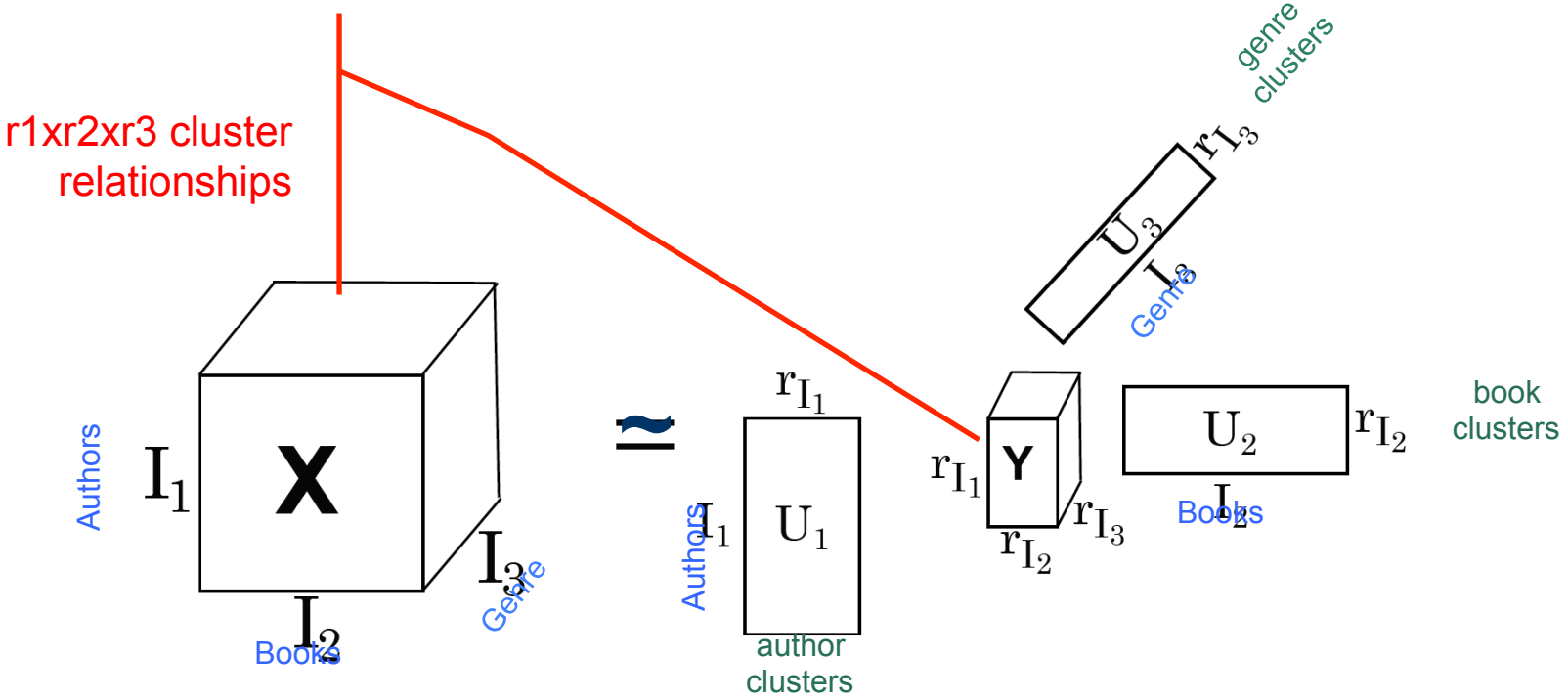

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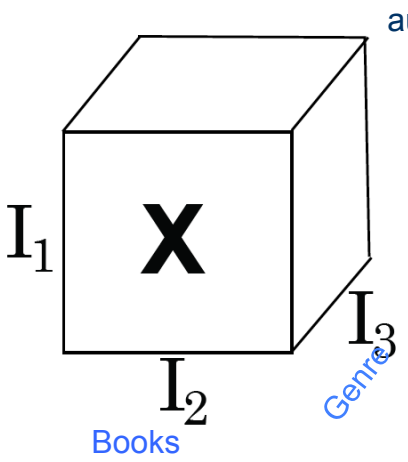
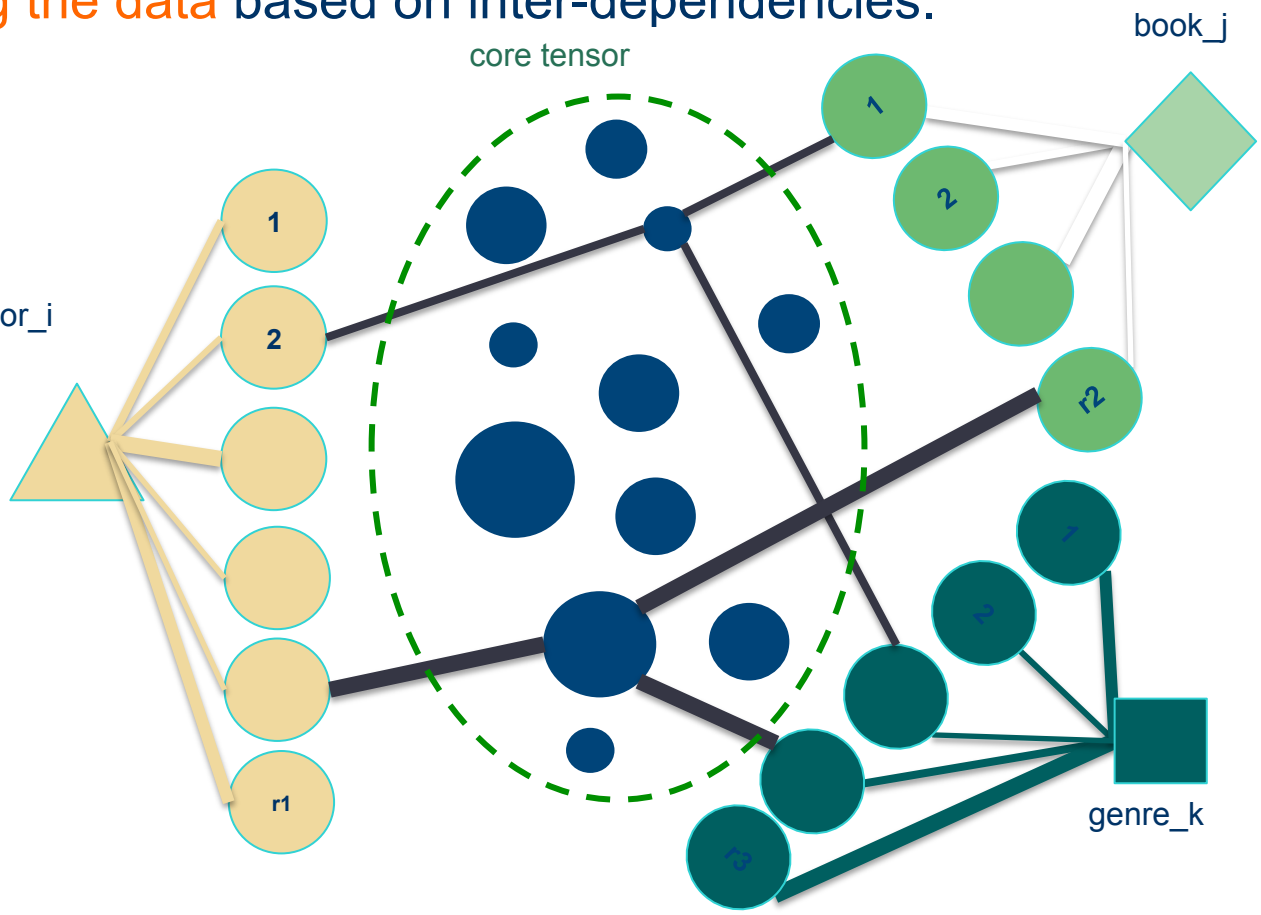
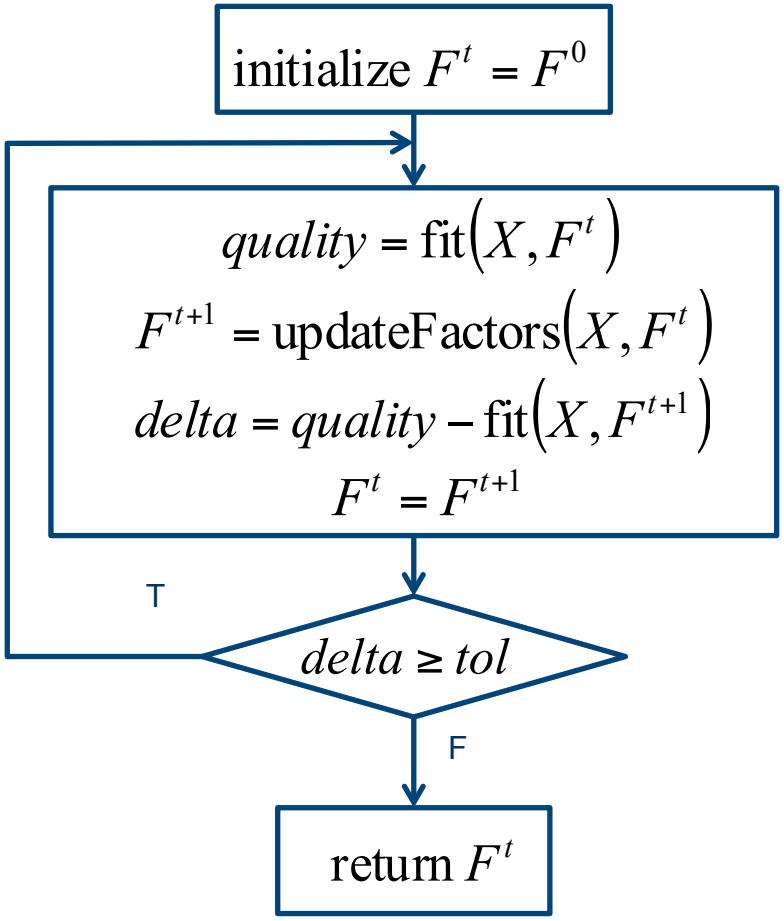




Fitting tensor decompositions

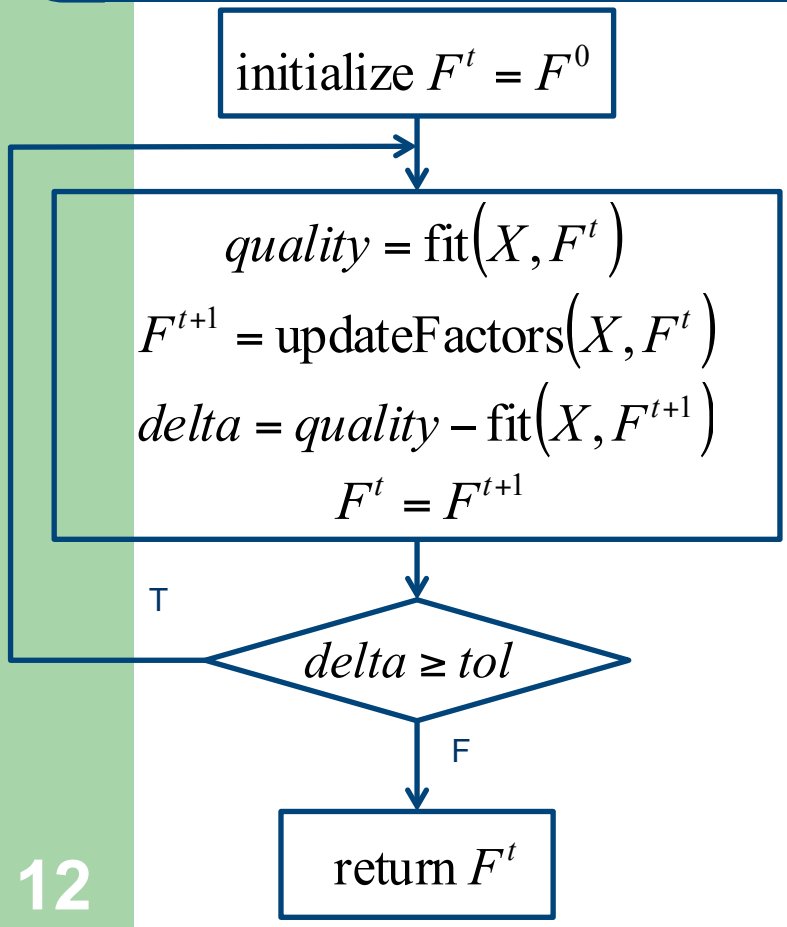
- Iterative algorithms
  - Alternating Least Squares (ALS)
  - Alternating Slice-Wise Diagonalization (ASD)
  - Self Weighted Alternating Trilinear Diagonalization (SWA-TLD)
- Closed form algorithms
  - Generalized rank annihilation method (GRAM)
  - Direct trilinear decomposition (DTLD)
- Gradient-based methods
  - PMF3 (based on Gauss-Newton method)

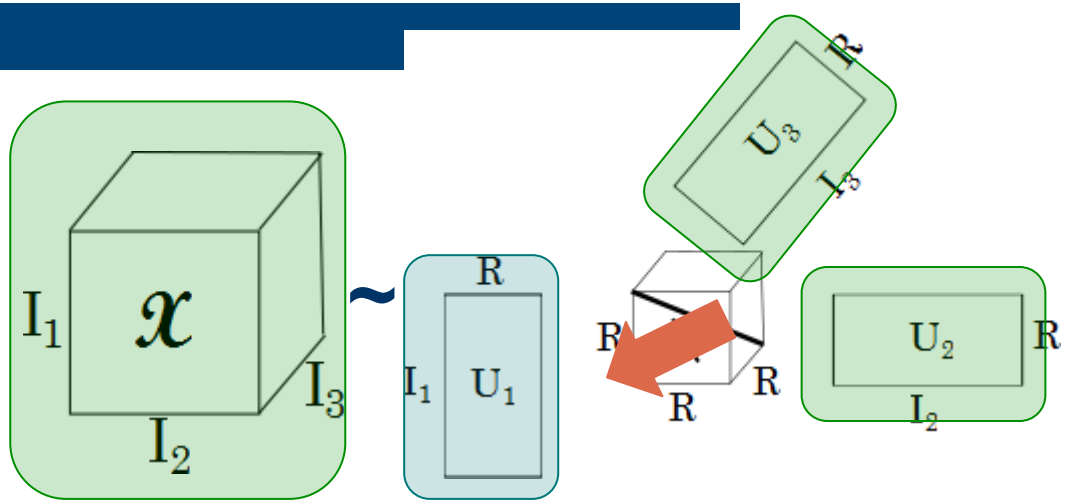
Iterative ALS algorithm



- At each step, all factor matrices are updated one at a time
- A factor matrix is estimated starting from the others

[Carrol et al., 1970
Harshman, 1970]





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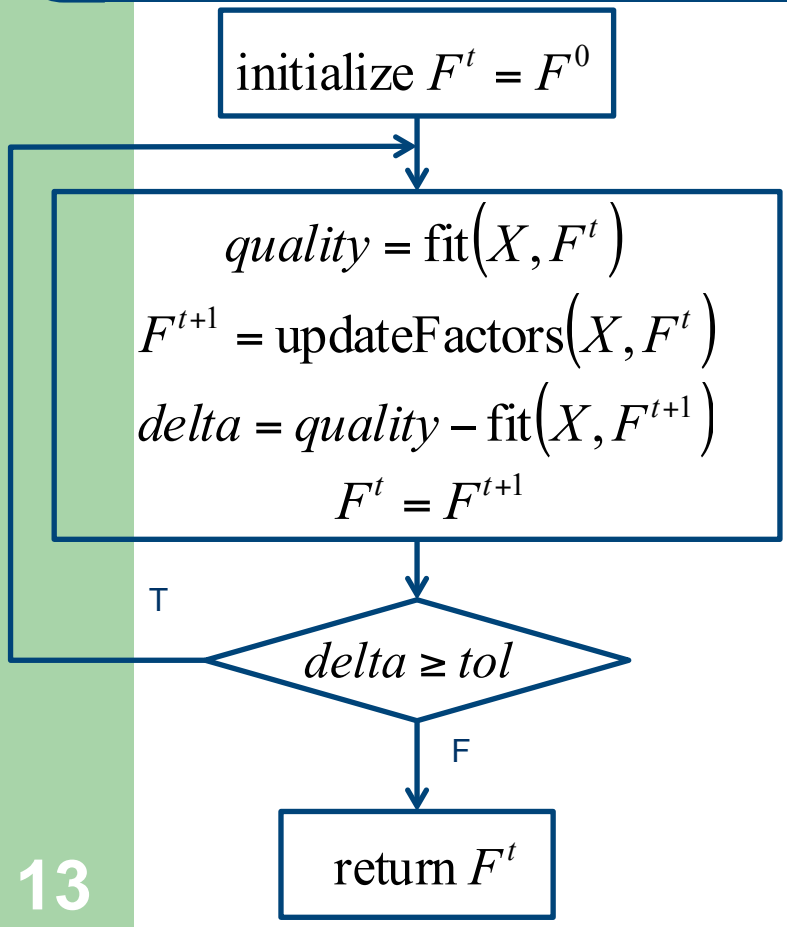

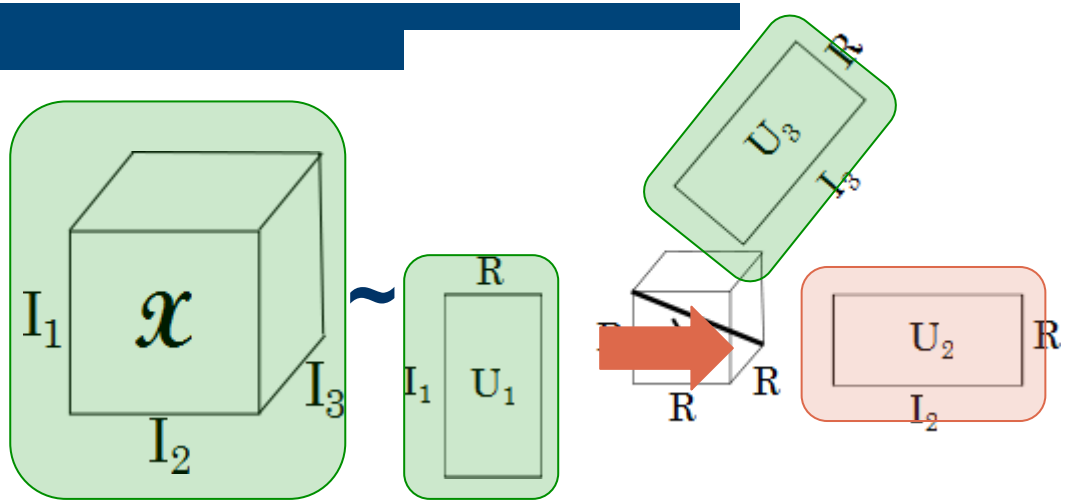

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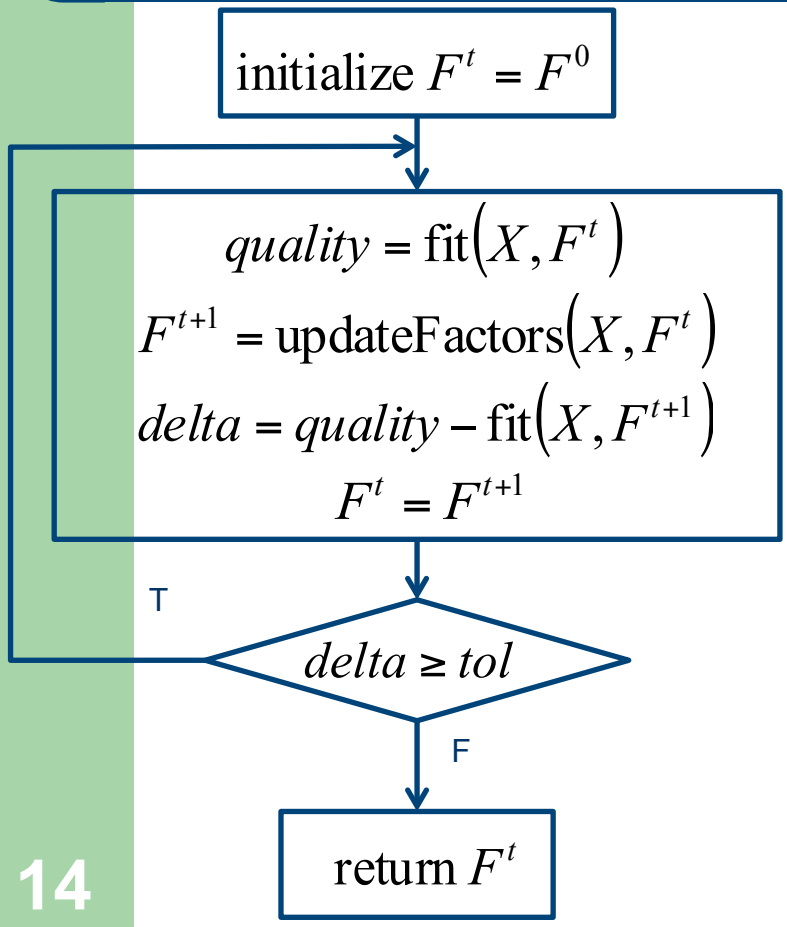
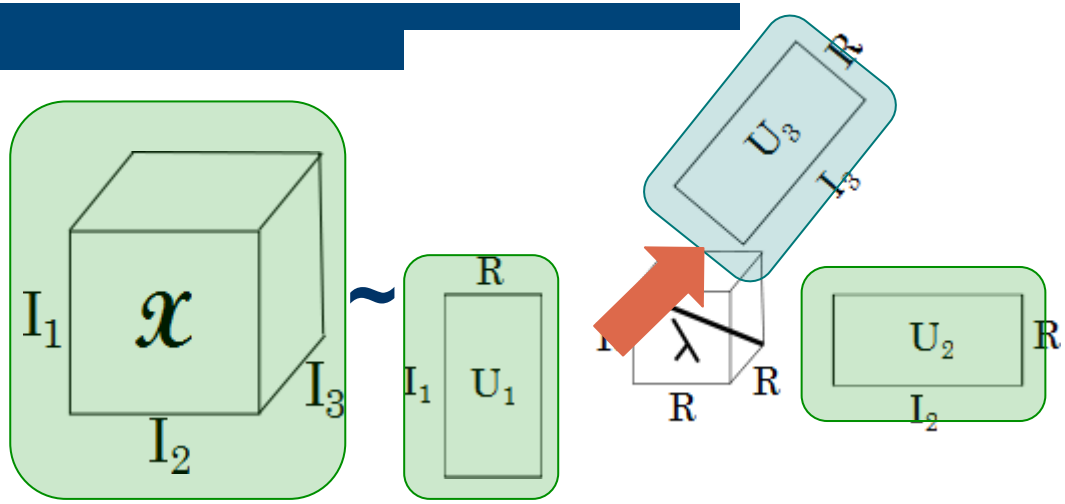
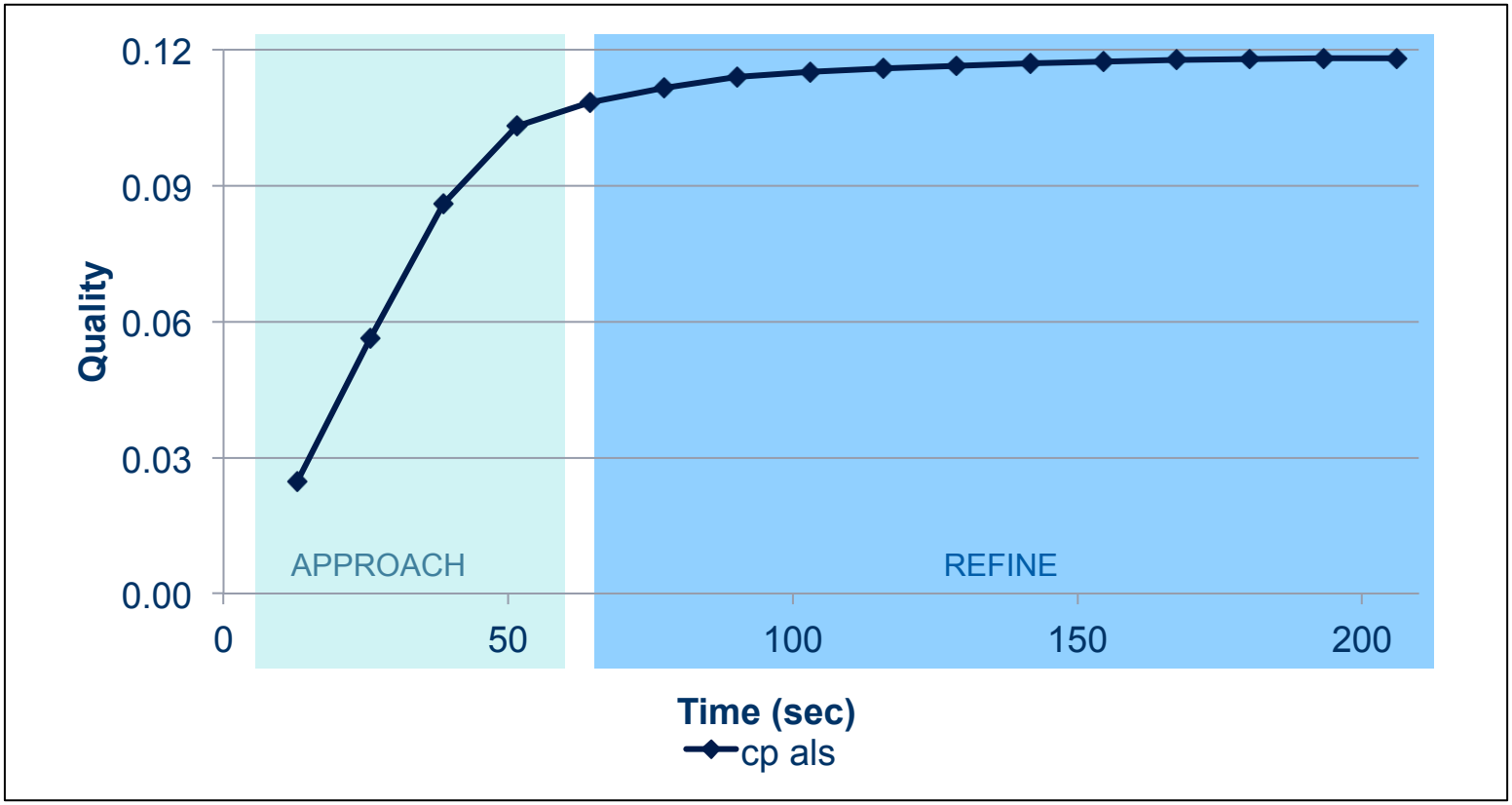




Iterative ALS



Observations

- Tensor decomposition algorithms are, especially for dense tensors, time consuming:
  - For dense tensors: exponential in the number of modes
  - For sparse tensors: linear in the number of non-zero elements.