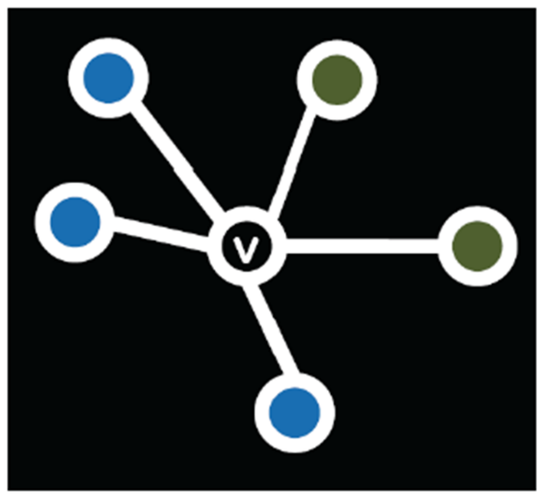
Cascading Behavior on Networks

Chapter 19

[Morris 2000]

Game Theoretic Model of Cascades

- **Based on 2 player coordination game**
  - 2 players – each chooses technology A or B
  - Each person can only adopt **one** "behavior", **A** or **B**
  - You gain more payoff if your friend has adopted the **same** behavior as you

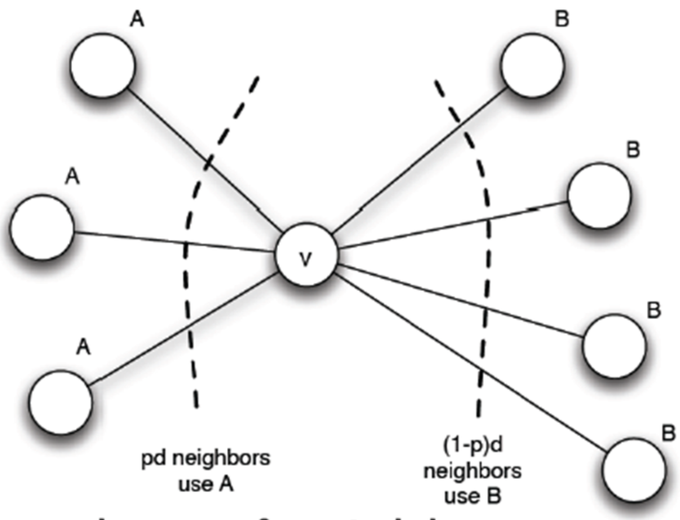


Local view of the network of node **v**

Example: VHS vs. BetaMax

Example: BlueRay vs. HD DVD

Calculation of Node v



Threshold:
v choses ***A*** if

$$p > \frac{b}{a+b} = q$$

p… frac. v's nbrs. with A
q… payoff threshold

- Let **v** have ***d*** neighbors
- Assume fraction ***p*** of ***v***'s neighbors adopt ***A***
  - $Payoff_v = a \cdot p \cdot d$ if v chooses A
    $= b \cdot (1-p) \cdot d$ if v chooses B
- **Thus:** ***v*** **chooses A if:** $a \cdot p \cdot d > b \cdot (1-p) \cdot d$

Example Scenario

Scenario:

- Graph where everyone starts with ***B***
- Small set ***S*** of early adopters of ***A***
  - Hard-wire ***S*** – they keep using ***A*** no matter what payoffs tell them to do

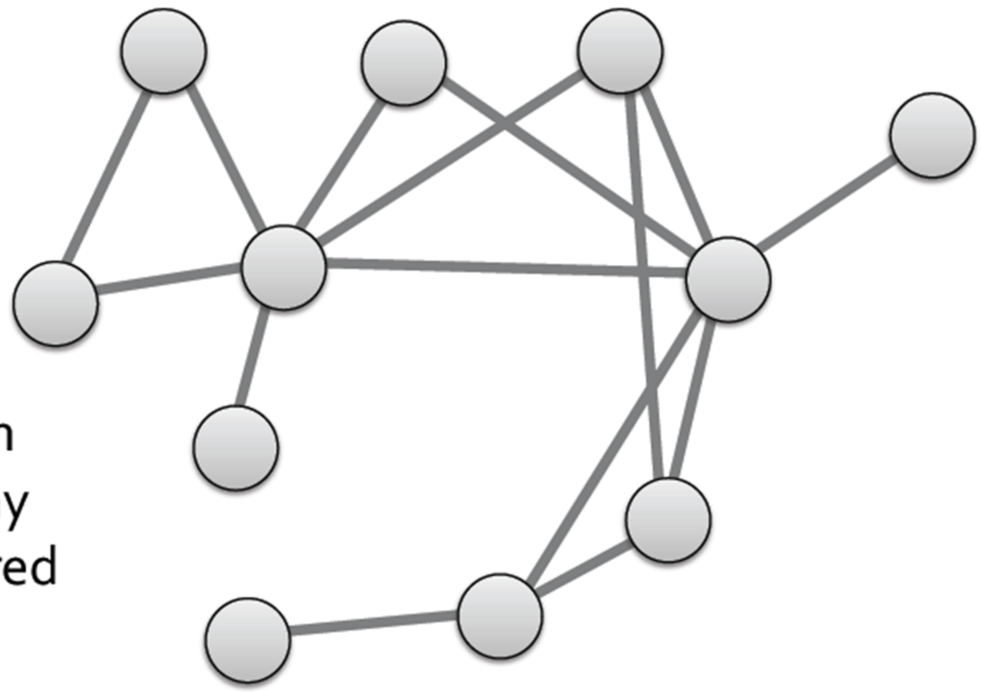
- **Assume payoffs are set in such a way that nodes say:**

  If more than 50% of my friends take A I'll also take A

  This means: **a = b-ε** (ε>0, small positive constant) and **q>1/2**

Example Scenario

$S = \{u, v\}$



If **more** than **q**=50% of my friends are red I'll be red

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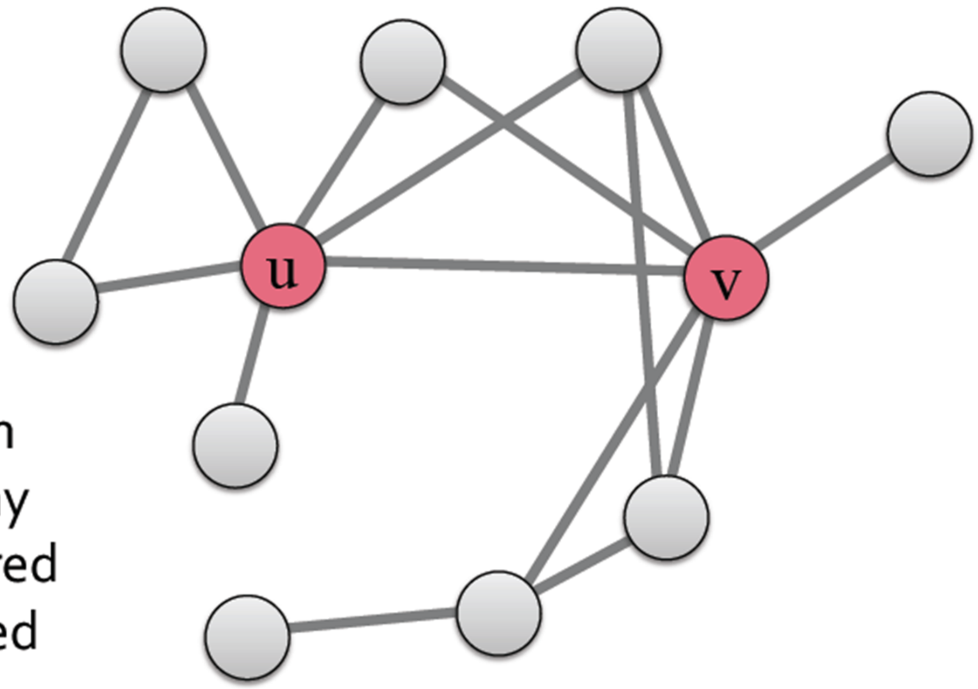

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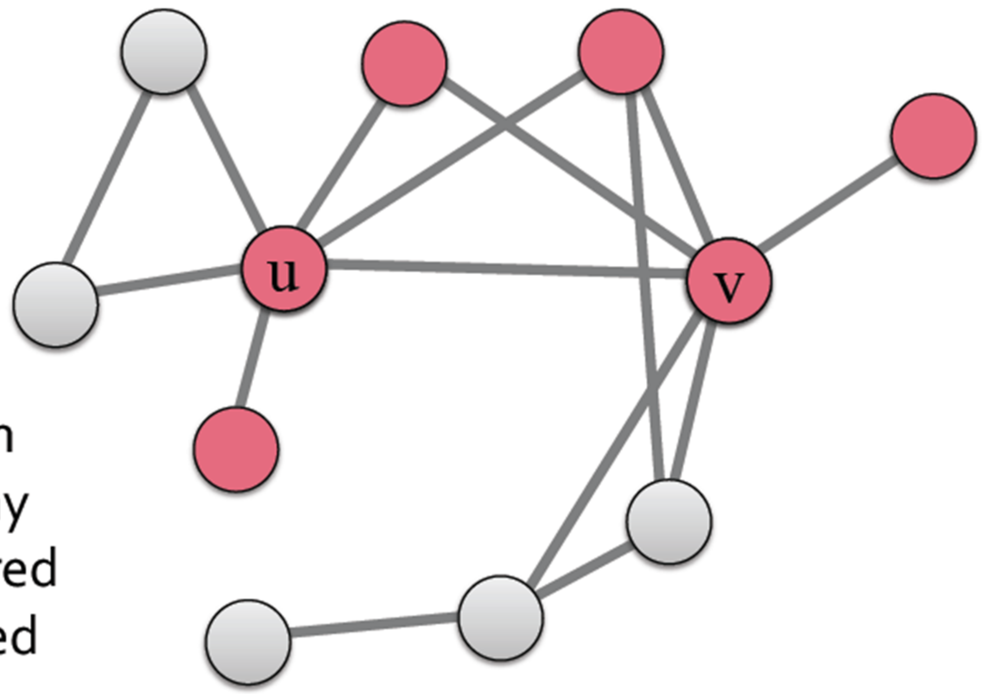

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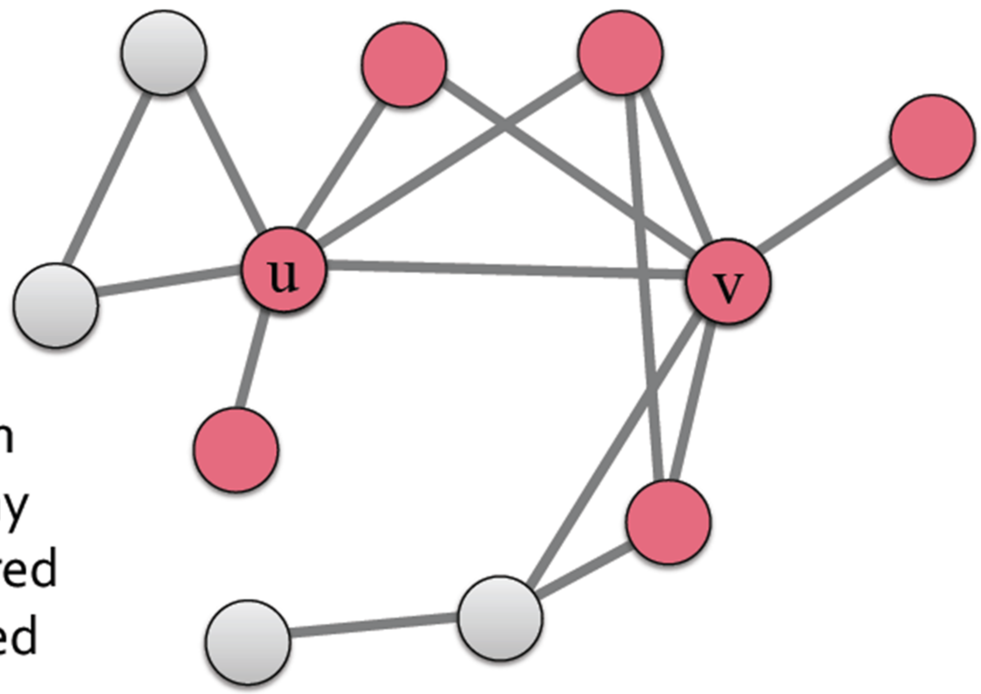

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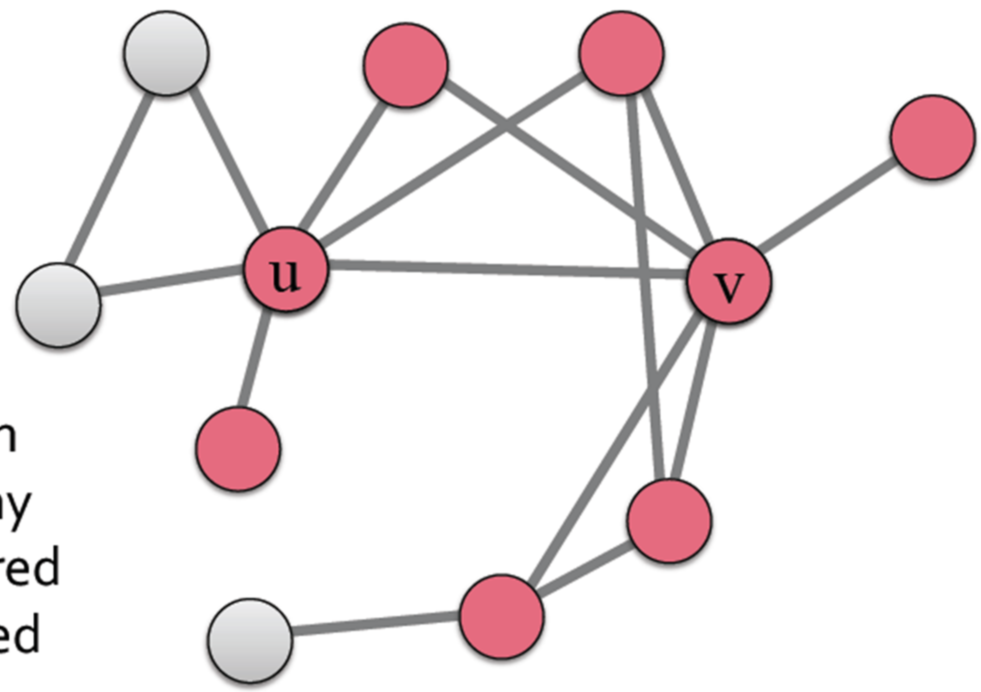

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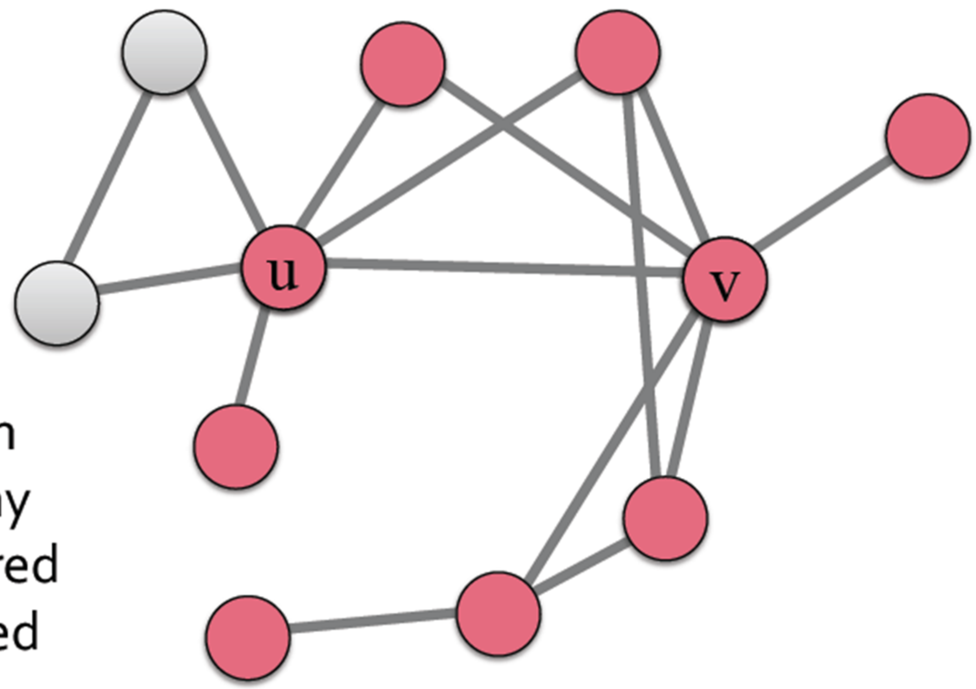

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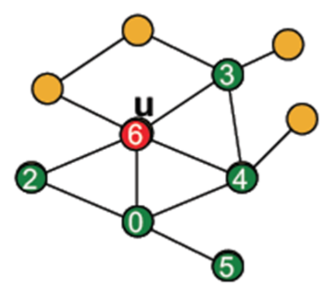
Monotonic Spreading

- **Observation: Use of A spreads monotonically**
  (Nodes only switch **B→A**, but never back to **B**)
- **Why?** Proof sketch:
  - **Nodes keep switching from B to A:** *B→A*
  - Now, suppose some node switched back from ***A→B***, consider the **first** node ***u*** (not in **S**) to do so (say at time ***t***)
  - Earlier at some time ***t'*** (***t'<t***) the same node ***u*** switched ***B→A***
  - So at time ***t' u*** was above threshold for ***A***
  - But up to time ***t*** no node switched back to **B**, so node ***u*** could only have more neighbors who used ***A*** at time ***t*** compared to ***t'***.
    There was no reason for *u* to switch at the first place!



!! Contradiction !!

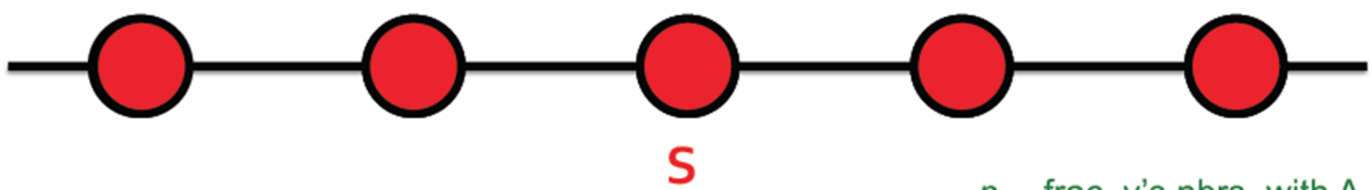
Infinite Graphs

v chooses *A* if *p>q*

$$q = \frac{b}{a+b}$$

- **Consider infinite graph *G***
  - (but each node has finite number of neighbors!)
- We say that a finite set ***S*** causes a cascade in *G* with **threshold *q*** if, when ***S*** adopts ***A***, eventually **every node in G adopts *A***
- Example: **Path**

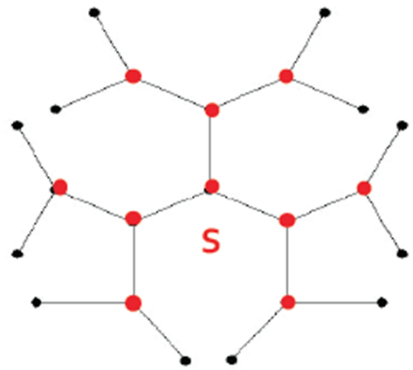
If ***q<1/2*** then cascade occurs



p… frac. v's nbrs. with A
q… payoff threshold

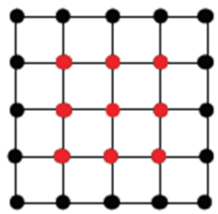
Infinite Graphs

- **Infinite Tree:**



If *q<1/3* then cascade occurs

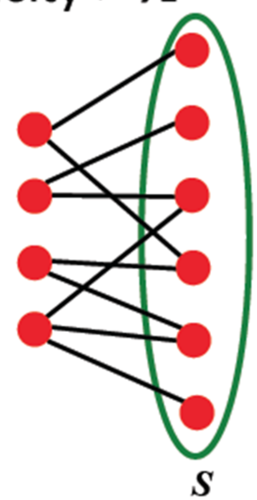
- **Infinite Grid:**



If *q<1/4* then cascade occurs

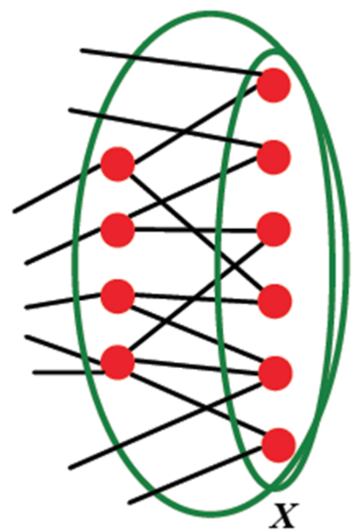
Cascade Capacity

- <u>Def:</u>
  - The **cascade capacity** of a graph ***G*** is the **largest *q*** for which some **finite set S** can cause a **cascade**
- <u>Fact:</u>
  - There is no (infinite) ***G*** where cascade capacity **> ½**
- **<u>Proof idea:</u>**
  - Suppose such **G** exists: **q>½**, finite **S** causes cascade
  - **Show contradiction:** Argue that nodes stop switching after a finite # of steps



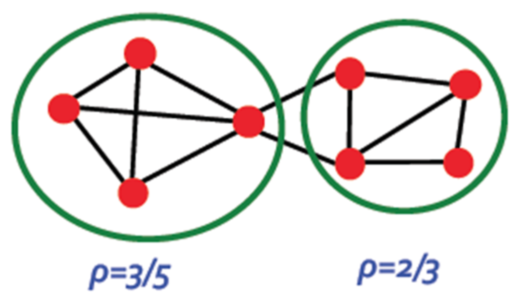
Cascade Capacity

- **<u>Fact:</u>** There is no ***G*** where cascade capacity **>** **½**
- **Proof sketch:**
  - Suppose such **G** exists: **q>½**, finite **S** causes cascade
  - **Contradiction:** Switching stops after a finite # of steps
    - <u>Define</u> "potential energy"
    - Argue that it starts finite (non-negative) and strictly decreases at every step
  - "Energy": = **$|d^{out}(X)|$**
    - **$|d^{out}(X)|$** := # of outgoing edges of active set *X*
    - The only nodes that switch have a strict majority of its neighbors in ***S***
    - **$|d^{out}(X)|$** strictly decreases
    - It can do so only for a finite number of steps



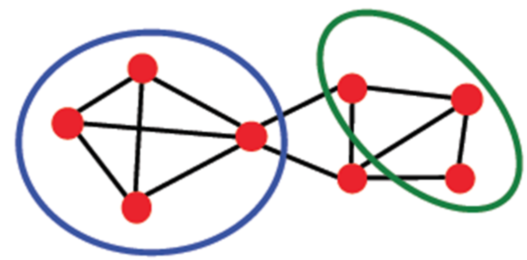
Stopping Cascades

- **What prevents cascades from spreading?**
- Def: **Cluster of density *ρ*** is a **set of nodes *C*** where **each node** in the set has at least ***ρ*** fraction of edges in ***C***



Stopping Cascades

- **Let S be an initial set of adopters of A**
- All nodes apply threshold ***q*** to decide whether to switch to **A**



- **Two facts:**
  - 1) If ***G\S*** contains a cluster of density ***>(1-q)*** then ***S*** cannot cause a cascade
  - 2) If **S** fails to create a cascade, then there is a cluster of density ***>(1-q)*** in **G\S**

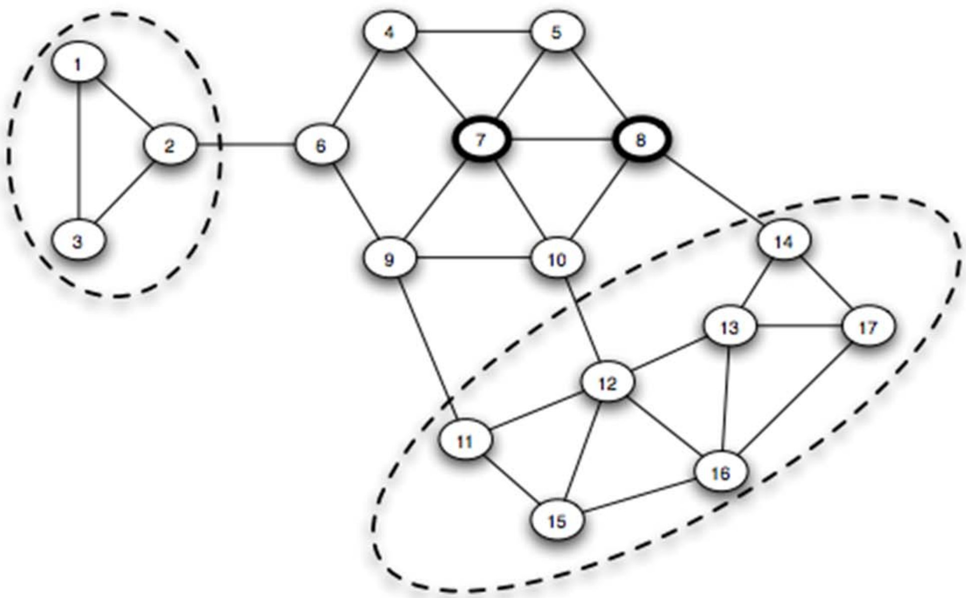


Figure 19.7

Claim: Consider a set of initial adopters of behavior A, with a threshold of q for nodes in the remaining network to adopt behavior A.

(i) If the remaining network contains a cluster of density greater than $1 - q$, then the set of initial adopters will not cause a complete cascade.

(ii) Moreover, whenever a set of initial adopters does not cause a complete cascade with threshold q, the remaining network must contain a cluster of density greater than $1 - q$.

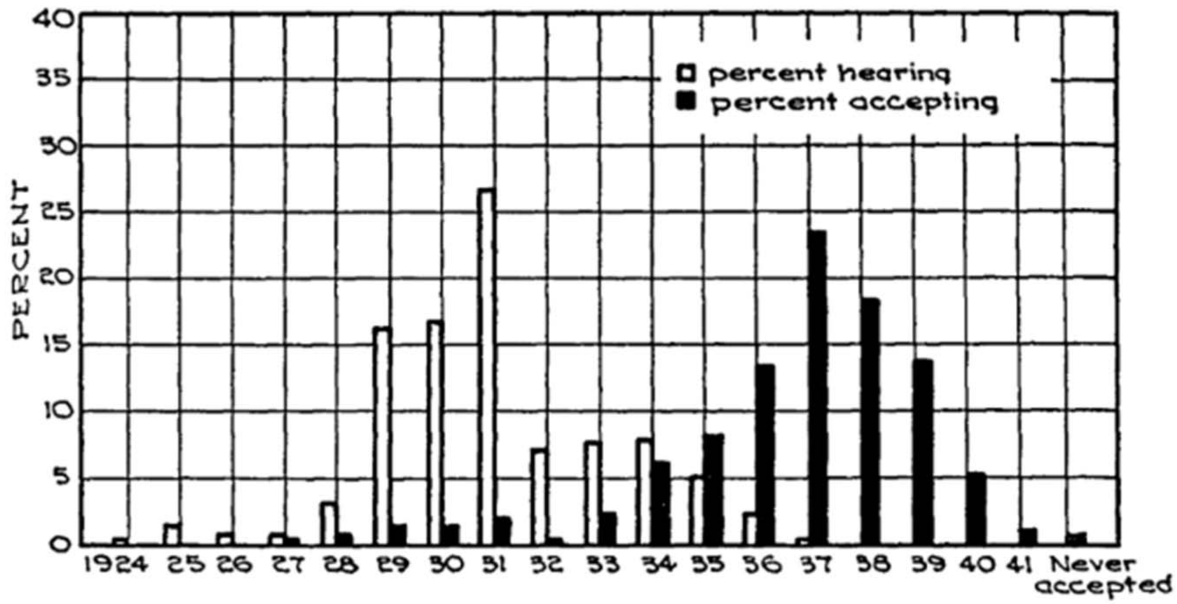


Figure 19.10: The years of first awareness and first adoption for hybrid seed corn in the Ryan-Gross study. (Image from [358].)

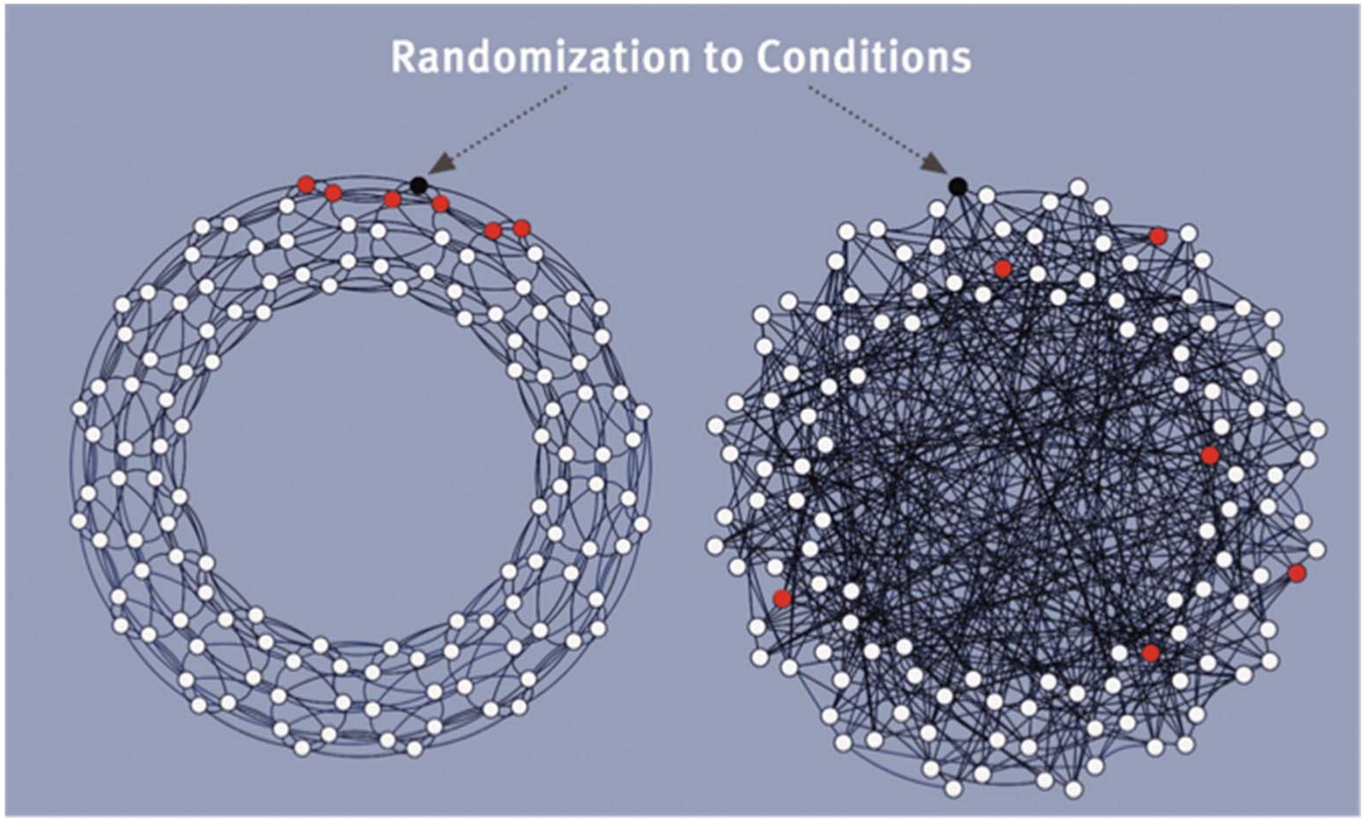
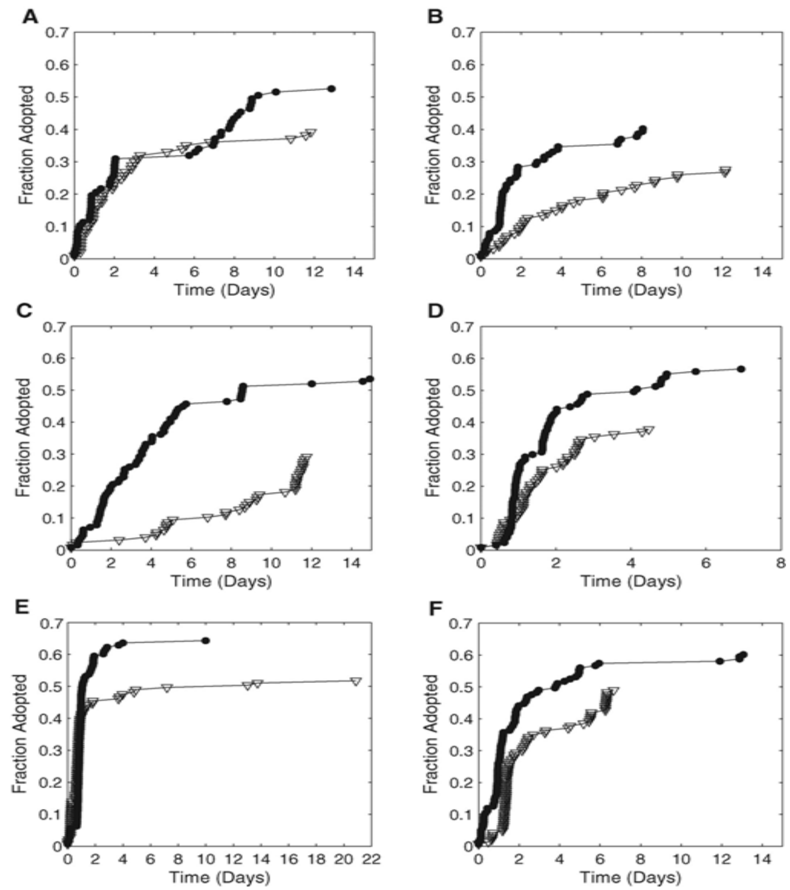
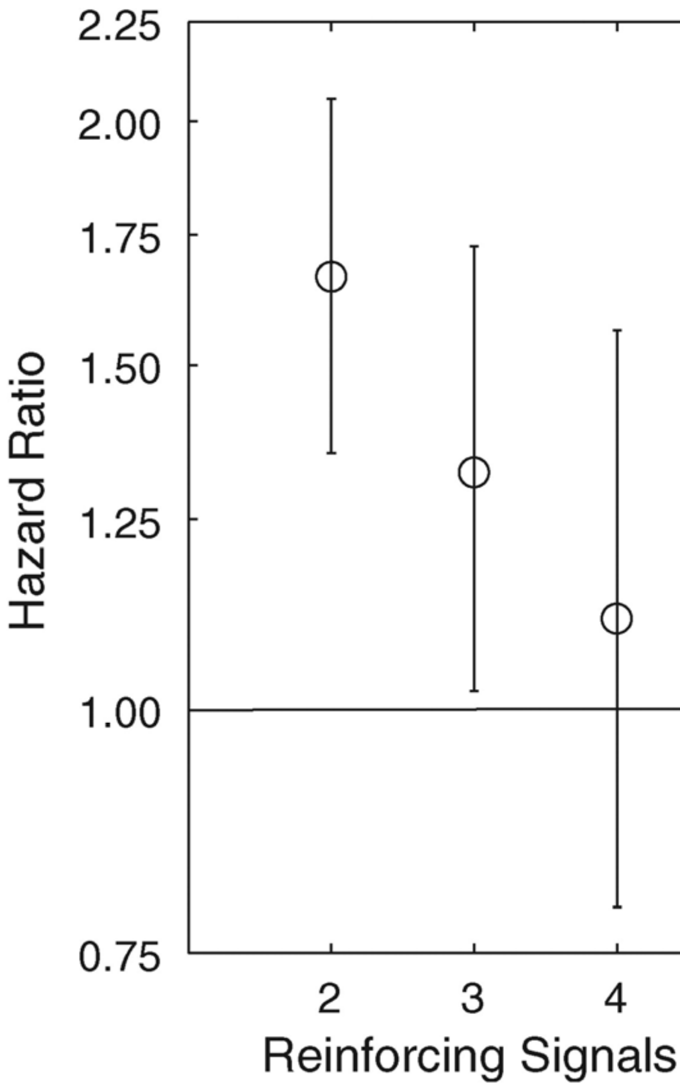
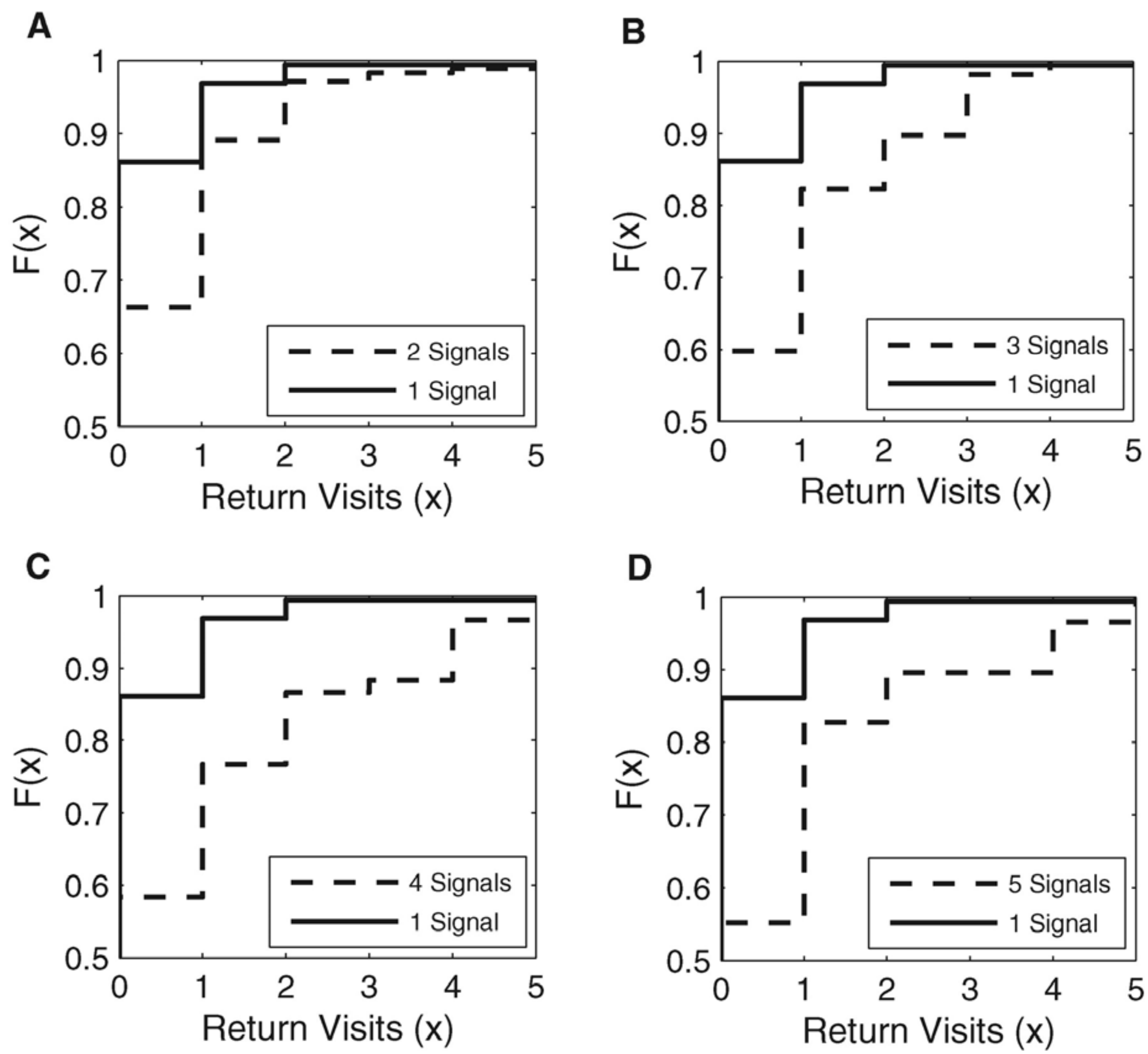

Randomization to Conditions

Fig. 2 Time series showing the adoption of a health behavior spreading through clustered-lattice (solid black circles) and random (open triangles) social networks.





2.25
2.00
1.75
1.50
1.25
1.00
0.75
Hazard Ratio
2
3
4
Reinforcing Signals


A
B
C
D
F(x)
Return Visits (x)
2 Signals
1 Signal
3 Signals
4 Signals
5 Signals
0.5
0.6
0.7
0.8
0.9
1
0
1
2
3
4
5

Extending the Model: Allow People to Adopt A and B