I am a professor in the School of Informatics, Computing and Engineering, and a member of the Center for Complex Networks and Systems Research at Indiana University, Bloomington. My background is in Statistical Mechanics, and my research interests are in the area of complex networks and their applications to the study of information diffusion, and computational social science.

Date	Topic
Wed, May 8th	Small World and Search (chapter 20 [1]; 20.1-20.2)
Thu, May 9th	Small World and Search (chapter 20 [1]; 20.1-20.2)
Mon, May 13th	Transportation Networks (chapter 8 [1])
Wed, May 15th	Information Cascades (chapter 16 [1]; 16.1-16.17)
Thu, May 16th	Network Effects (chapter 17 [1]; 17.1-17.17)
Mon, May 20th	Epidemics (chapter 21 [1]; 21.1,21.2)
Wed, May 22nd	Epidemics (chapter 21 [1]; 21.3, 21.8; paper [9])
Thu, May 23rd	Cascading Behavior in Networks (chapter 19 [1]; 19.1-19.7)

typically slower than $\sim 1\,\mathrm{km\,s^{-1}}$) might differ significantly from what is assumed by current modelling efforts[27]. The expected equation-of-state differences among small bodies (ice versus rock, for instance) presents another dimension of study; having recently adapted our code for massively parallel architectures (K. M. Olson and E.A, manuscript in preparation), we are now ready to perform a more comprehensive analysis.

The exploratory simulations presented here suggest that when a young, non-porous asteroid (if such exist) suffers extensive impact damage, the resulting fracture pattern largely defines the asteroid's response to future impacts. The stochastic nature of collisions implies that small asteroid interiors may be as diverse as their shapes and spin states. Detailed numerical simulations of impacts, using accurate shape models and rheologies, could shed light on how asteroid collisional response depends on internal configuration and shape, and hence on how planetesimals evolve. Detailed simulations are also required before one can predict the quantitative effects of nuclear explosions on Earth-crossing comets and asteroids, either for hazard mitigation[28] through disruption and deflection, or for resource exploitation[29]. Such predictions would require detailed reconnaissance concerning the composition and internal structure of the targeted object. □

Received 4 February; accepted 18 March 1998.

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11. Hudson, R. S. & Ostro, S. J. Shape of asteroid 4769 Castalia (1989 PB) from inversion of radar images.

Collective dynamics of 'small-world' networks

Duncan J. Watts* **& Steven H. Strogatz**

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Networks of coupled dynamical systems have been used to model biological oscillators[1–4], Josephson junction arrays[5,6], excitable media[7], neural networks[8–10], spatial games[11], genetic control networks[12] and many other self-organizing systems. Ordinarily, the connection topology is assumed to be either completely regular or completely random. But many biological, technological and social networks lie somewhere between these two extremes. Here we explore simple models of networks that can be tuned through this middle ground: regular networks 'rewired' to introduce increasing amounts of disorder. We find that these systems can be highly clustered, like regular lattices, yet have small characteristic path lengths, like random graphs. We call them 'small-world' networks, by analogy with the small-world phenomenon[13,14] (popularly known as six degrees of separation[15]). The neural network of the worm *Caenorhabditis elegans*, the power grid of the western United States, and the collaboration graph of film actors are shown to be small-world networks. Models of dynamical systems with small-world coupling display enhanced signal-propagation speed, computational power, and synchronizability. In particular, infectious diseases spread more easily in small-world networks than in regular lattices.

To interpolate between regular and random networks, we consider the following random rewiring procedure (Fig. 1). Starting from a ring lattice with n vertices and k edges per vertex, we rewire each edge at random with probability p. This construction allows us to 'tune' the graph between regularity ($p = 0$) and disorder ($p = 1$), and thereby to probe the intermediate region $0 < p < 1$, about which little is known.

“SMALL WORLD” PHENOMENON

Whose idea was it?

Milgram's experiment

- Stanley Milgram, psychologist at Harvard (famous for another exp.)
- 1967 experiment to measure "social distance" between any 2 people in the US
  - First idea was in the short story "Chains" by Hungarian writer Frigyes Karinthy in 1929
  - John Guare's 1991 play coined the term "six degrees of separation" (movie, too)

The Milgram's experiment

Did it work?

Milgram's experiment

* 42 letters made it back (only 26%)
* Range: 3-12 steps
* Average: 5.5 intermediates (6.5 steps)
* Much lower than most people expected!
* "Small world" effect is still surprising
* Half of letters arrived via 3 friends of target: "gatekeepers"?

Results

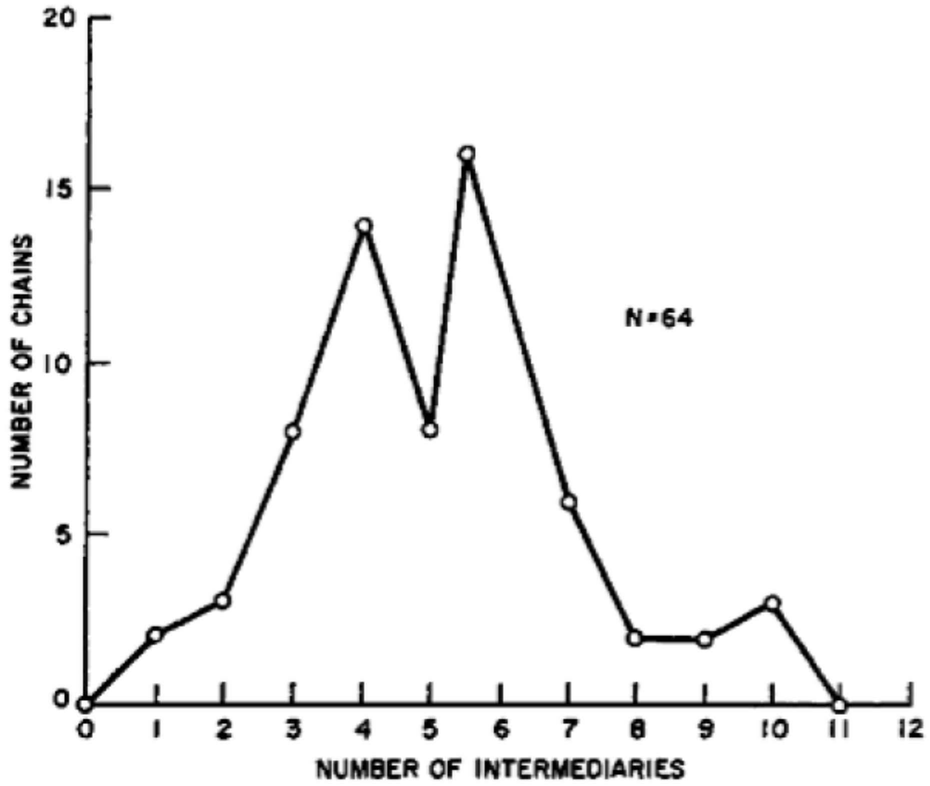
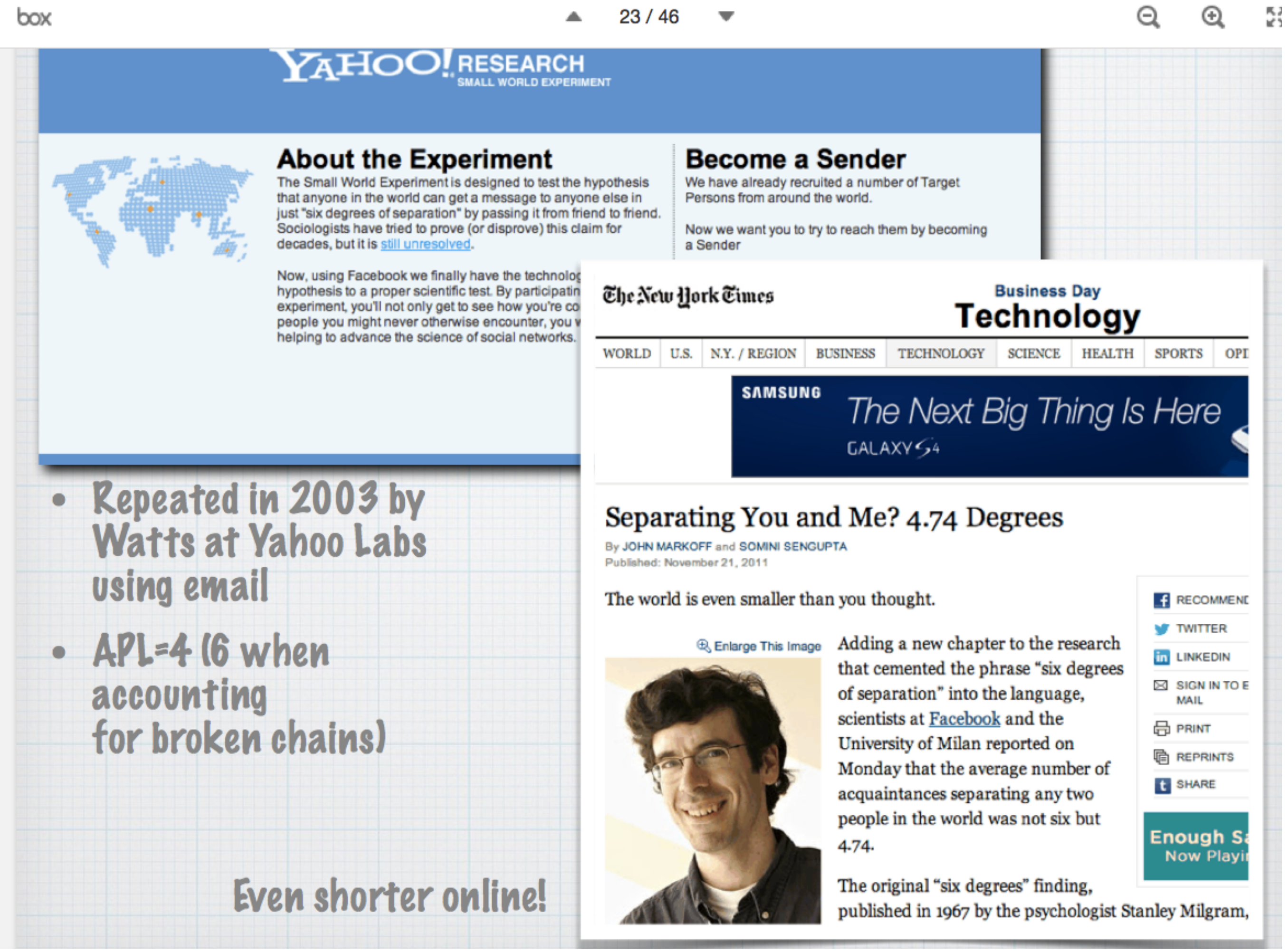
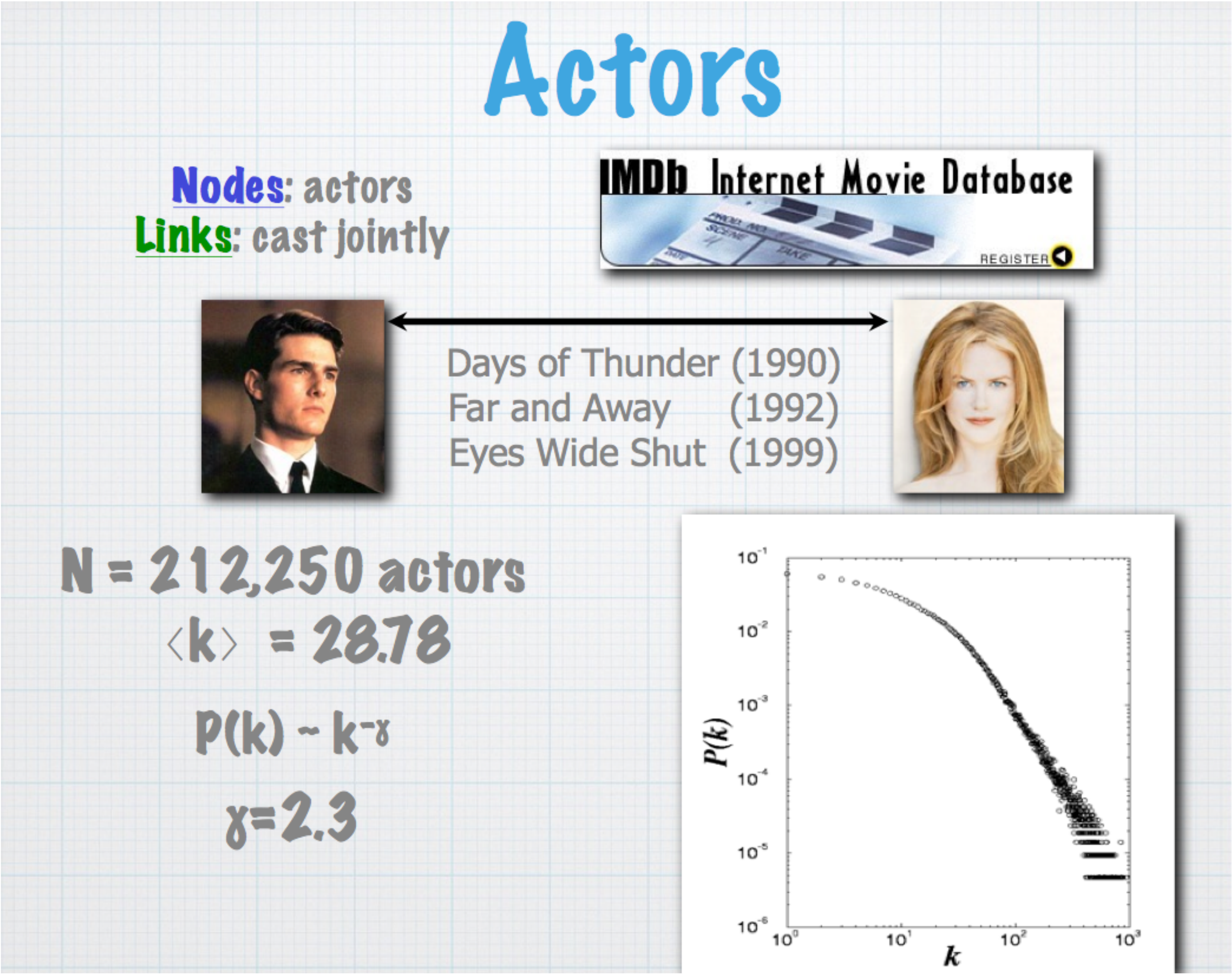


Figure 2.10: A histogram from Travers and Milgram's paper on their small-world experiment [391]. For each possible length (labeled "number of intermediaries" on the x-axis), the plot shows the number of successfully completed chains of that length. In total, 64 chains reached the target person, with a median length of six.

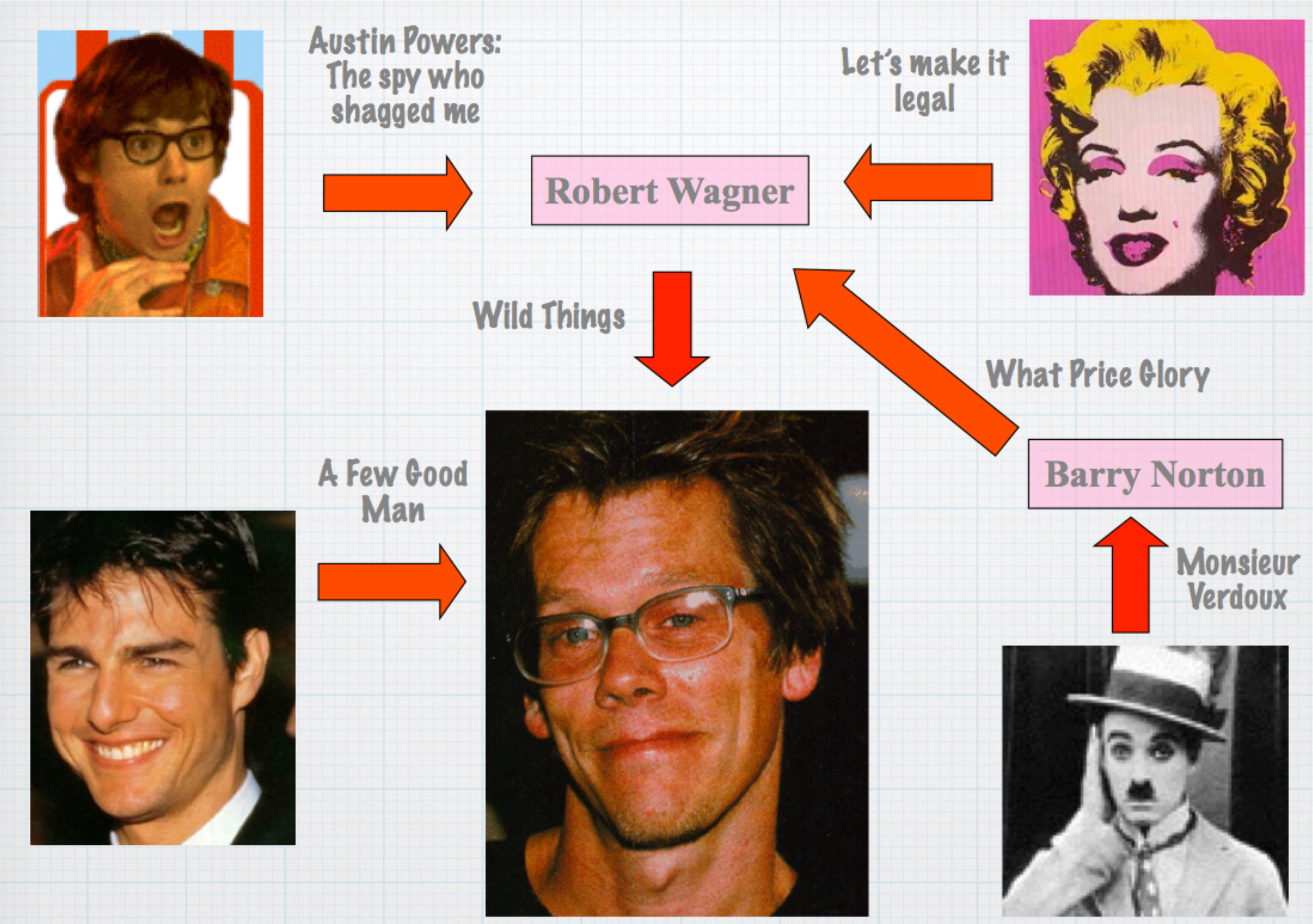
Replicating the Experiment



Toy datasets



https://oracleofbacon.org/



https://www.ted.com/talks/nicholas_christakis_the_hidden_influence_of_social_networks

Mike Pence
Governor of Indiana

Barack Obama
President of USA

Mike MC Robbie
President of IU

John Key,
NZ Prime Minister

Nick Smith
Congressman from Nelson, NZ

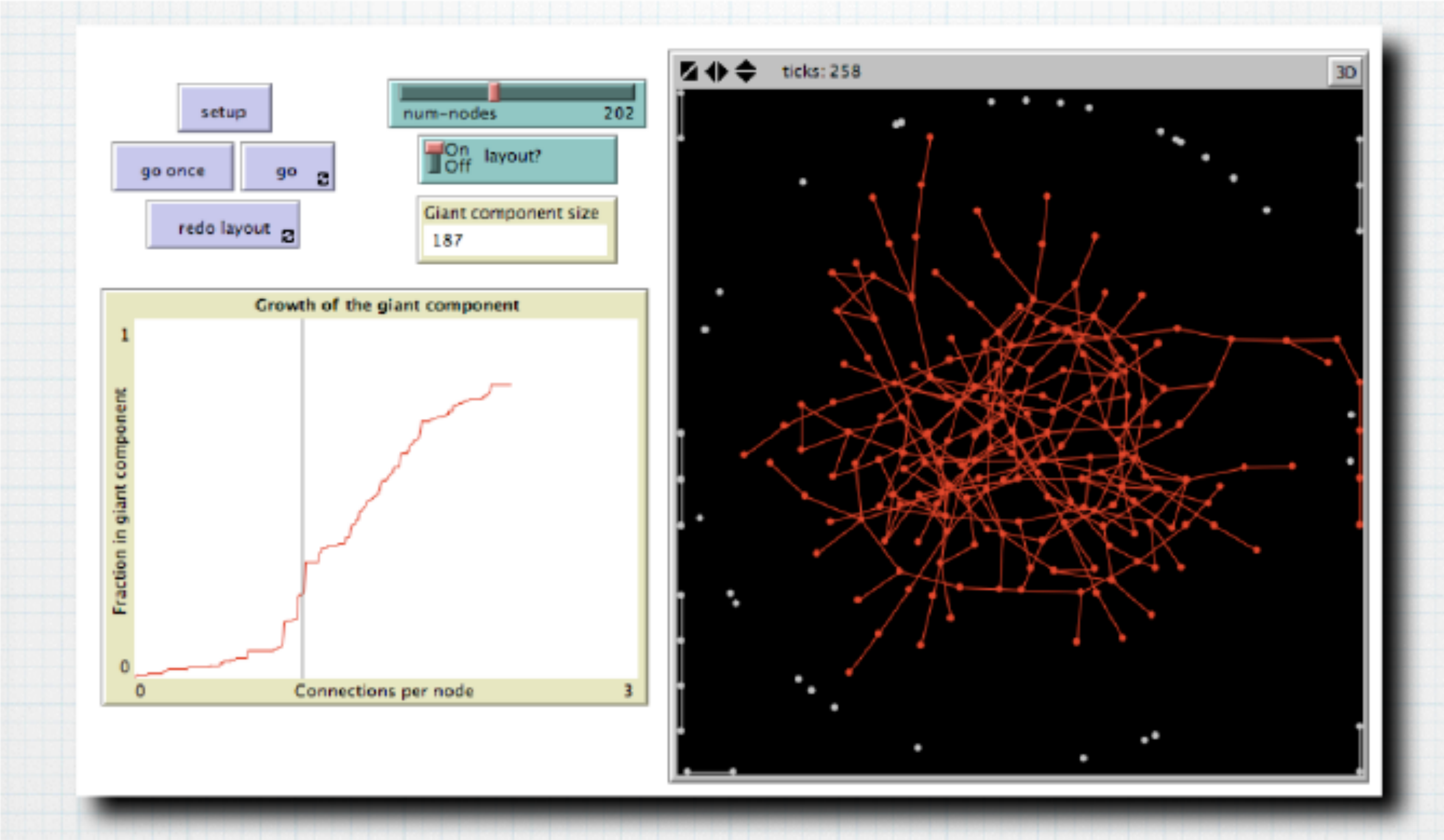
Brad Wheeler
Dean of Soic

Rachel Reese
Mayor of Nelson, NZ

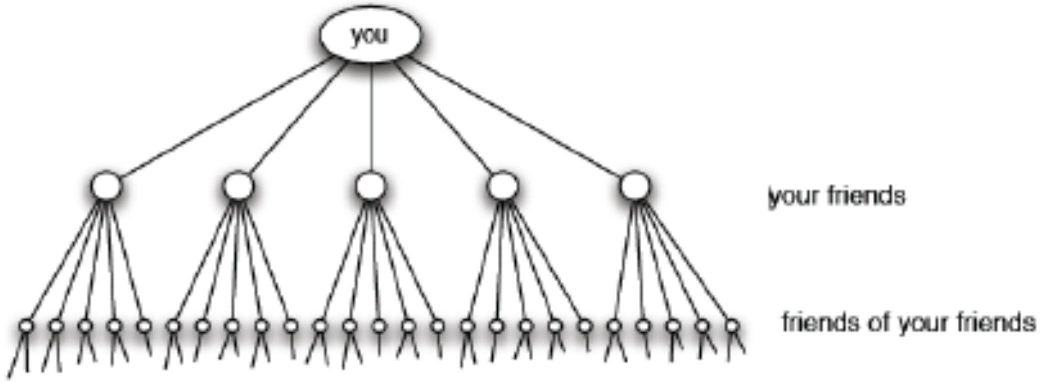
Alex

Mike Kirkpatrick
Fisherman in Nelson, NZ

How hard is to be connected?



How close are we?



(a) *Pure exponential growth produces a small world*

From: Networks, Crows and Markets by Easley and Kleinberg

Chapter 20, Q1, Easley and Kleinberg

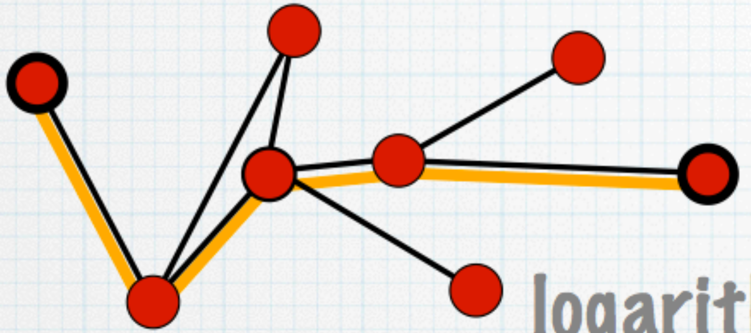
In the basic “six degrees of separation" question, one asks whether most pairs of people in the world are connected by a path of at most six edges in the social network, where an edge joins any two people who know each other on a first-name basis.

Now let's consider a variation on this question. Suppose that we consider the full population of the world, and suppose that from each person in the world we create a directed edge only to their ten closest friends (but not to anyone else they know on a first-name basis). In the resulting “closest-friend" version of the social network, is it possible that for each pair of people in the world, there is a path of at most six edges connecting this pair of people?

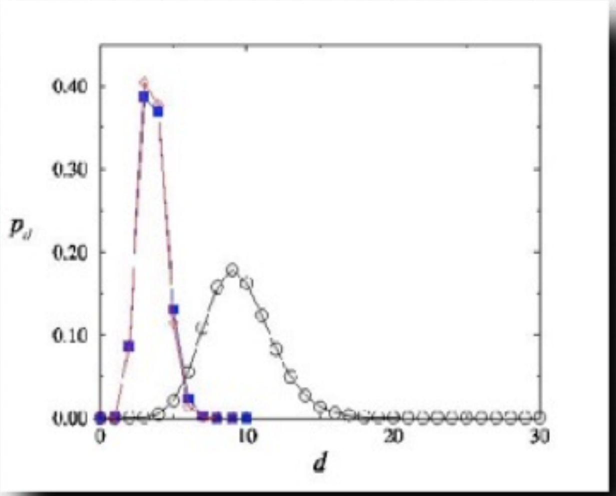
Explain.

Our plan

☑ Paths and average distance



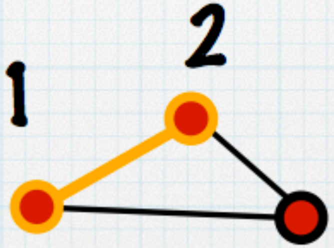
$$\langle \ell \rangle \simeq \frac{\log N}{\log \langle k \rangle}$$



logarithmically small

☐ Clustering coefficient

What is the probability that nodes 1 and 2 are linked?



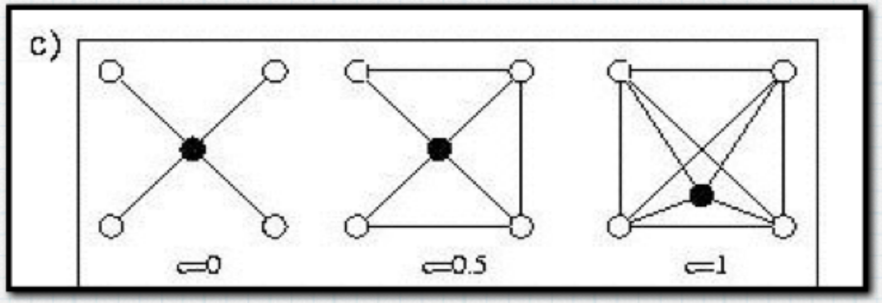
C = p = <k>/(N-1)

vanishingly small for large network sizes

Clustering Coefficient

- What portion of your neighbors are connected?
  - Node i with degree k_i

$$C_i = \frac{2e_i}{k_i(k_i - 1)}$$

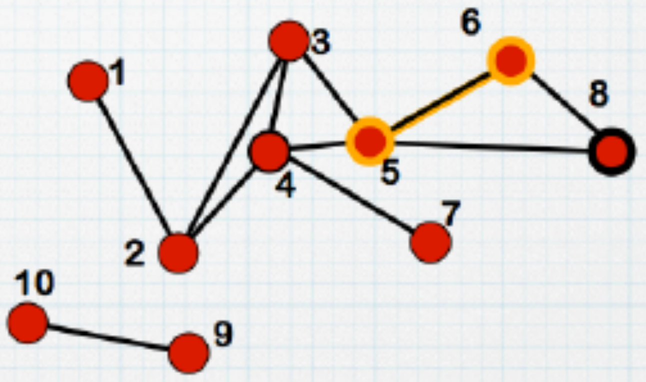


  - C_i in [0,1]

Our plan

- What portion of your neighbors are connected?
  - Node i with degree k_i

$$C_i = \frac{2e_i}{k_i(k_i - 1)}$$



$i=8$: $k_8=2$, $e_8=1$, $TOT=2*1/2=1$ → $C_8=1/1=1$

Can random models explain *l* & *cc*?

Network	Size	$\langle k \rangle$	ℓ	ℓ_{rand}	C	C_{rand}	Reference	Nr.
WWW, site level, undir.	153 127	35.21	3.1	3.35	0.1078	0.00023	Adamic, 1999	1
Internet, domain level	3015–6209	3.52–4.11	3.7–3.76	6.36–6.18	0.18–0.3	0.001	Yook *et al.*, 2001a, Pastor-Satorras *et al.*, 2001	2
Movie actors	225 226	61	3.65	2.99	0.79	0.00027	Watts and Strogatz, 1998	3
LANL co-authorship	52 909	9.7	5.9	4.79	0.43	1.8×10^{-4}	Newman, 2001a, 2001b, 2001c	4
MEDLINE co-authorship	1 520 251	18.1	4.6	4.91	0.066	1.1×10^{-5}	Newman, 2001a, 2001b, 2001c	5
SPIRES co-authorship	56 627	173	4.0	2.12	0.726	0.003	Newman, 2001a, 2001b, 2001c	6
NCSTRL co-authorship	11 994	3.59	9.7	7.34	0.496	3×10^{-4}	Newman, 2001a, 2001b, 2001c	7
Math. co-authorship	70 975	3.9	9.5	8.2	0.59	5.4×10^{-5}	Barabási *et al.*, 2001	8
Neurosci. co-authorship	209 293	11.5	6	5.01	0.76	5.5×10^{-5}	Barabási *et al.*, 2001	9
E. coli, substrate graph	282	7.35	2.9	3.04	0.32	0.026	Wagner and Fell, 2000	10
E. coli, reaction graph	315	28.3	2.62	1.98	0.59	0.09	Wagner and Fell, 2000	11
Ythan estuary food web	134	8.7	2.43	2.26	0.22	0.06	Montoya and Solé, 2000	12
Silwood Park food web	154	4.75	3.40	3.23	0.15	0.03	Montoya and Solé, 2000	13
Words, co-occurrence	460.902	70.13	2.67	3.03	0.437	0.0001	Ferrer i Cancho and Solé, 2001	14
Words, synonyms	22 311	13.48	4.5	3.84	0.7	0.0006	Yook *et al.*, 2001b	15
Power grid	4941	2.67	18.7	12.4	0.08	0.005	Watts and Strogatz, 1998	16
C. Elegans	282	14	2.65	2.25	0.28	0.05	Watts and Strogatz, 1998	17

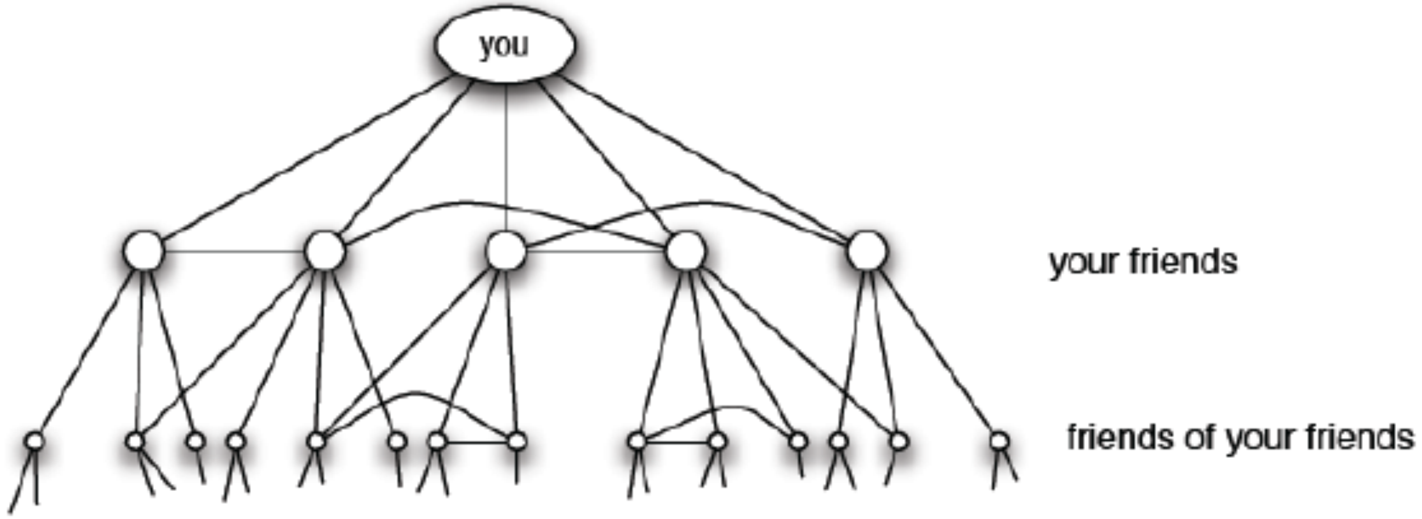
UNIVERSITY OF CAMBRIDGE

From: https://www.cl.cam.ac.uk/teaching/1213/L109/stna-lecture2.pdf

What is missing?

- Social (and many other) networks have both short paths and high clustering
- Random networks have short paths but not high clustering
  - If coauthorship networks were random graphs, they would have: $C = p = \langle k \rangle/(N-1) \approx 10^{-5} \ll 0.5$
- So, how can we reproduce both?

Is this really a good explanation?



(b) *Triadic closure reduces the growth rate*

From: Networks, Crows and Markets by Easley and Kleinberg

Chapter 20, Q2, Easley and Kleinberg

Now let's consider a variation on the “six degree” question. For each person in the world, we ask them to rank the 30 people they know best, in descending order of how well they know them. (Let's suppose for purposes of this question that each person is able to think of 30 people to list.) We then construct two different social networks:

(a) The “close-friend" network: from each person we create a directed edge only to their ten closest friends on the list.

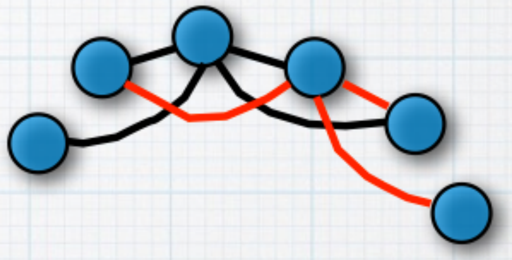
(b) The \distant-friend" network: from each person we create a directed edge only to the ten people listed in positions 21 through 30 on their list.

Let's think about how the small-world phenomenon might differ in these two networks. In particular, let C be the average number of people that a person can reach in six steps in the close-friend network, and let D be the average number of people that a person can reach in six steps in the distant-friend network (taking the average over all people in the world).

When researchers have done empirical studies to compare these two types of networks (the exact details often differ from one study to another), they tend to find that one of C or D is consistently larger than the other.

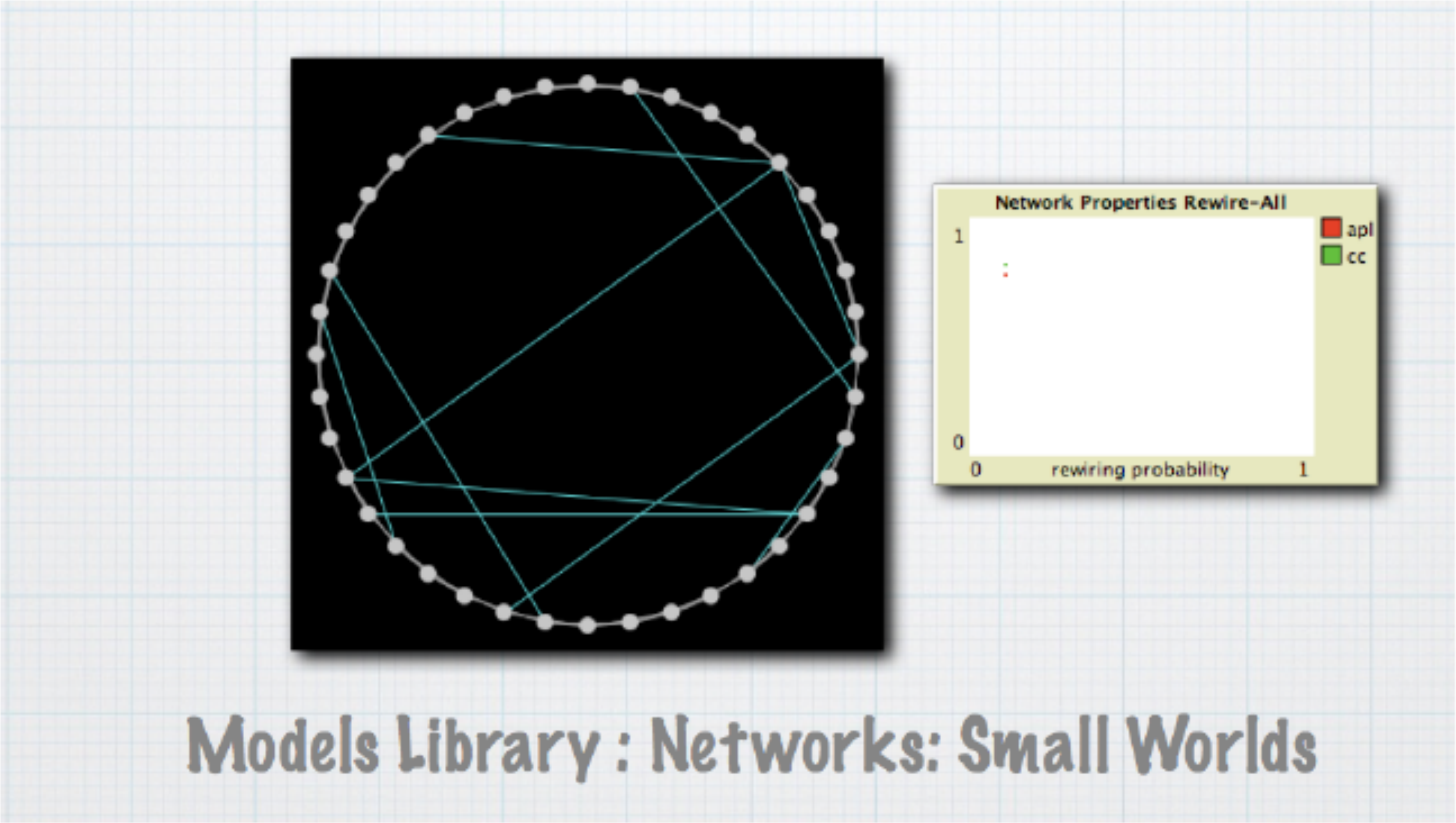
Which of the two quantities, C or D, do you expect to be larger? Give a brief explanation for your answer.

Setting of the W&S model



- Start with a “regular network” (lattice)
  - Each node linked to, say, 4 neighbors on a line
  - High clustering coefficient: C=?
- Pick a random link and rewire it at random with probability p; repeat...

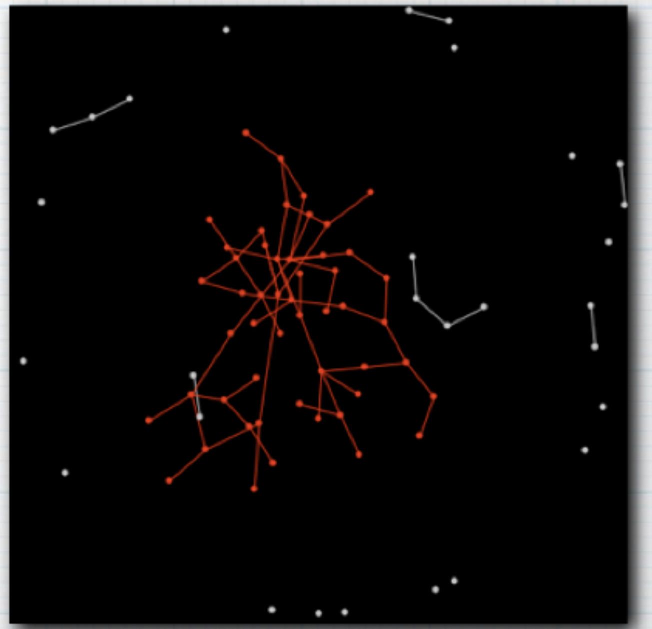
Netlogo


Network Properties Rewire-All
1
0
apl
cc
0
rewiring probability
1
Models Library : Networks: Small Worlds

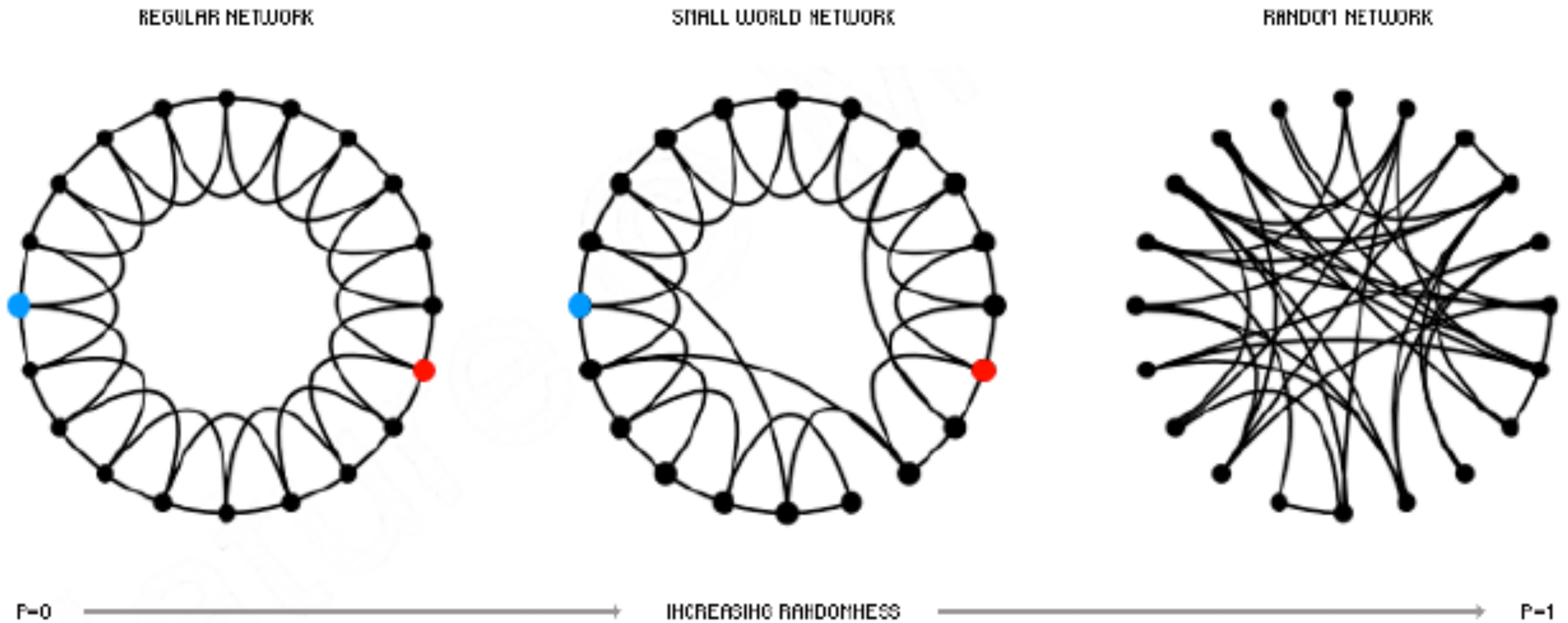
Watts & Strogatz

- Two properties:
  1. "small" diameter or apl
  2. "high" clustering coefficient

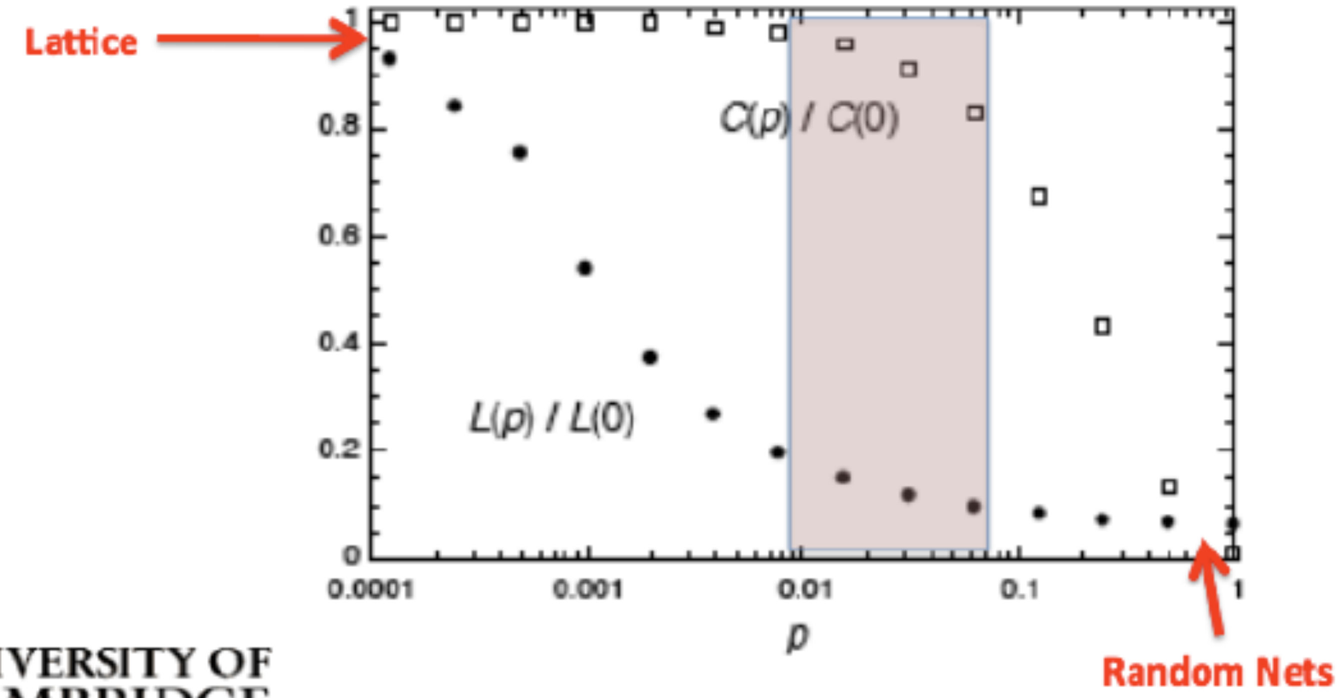
- Based on this definition, are random graphs "small worlds"?



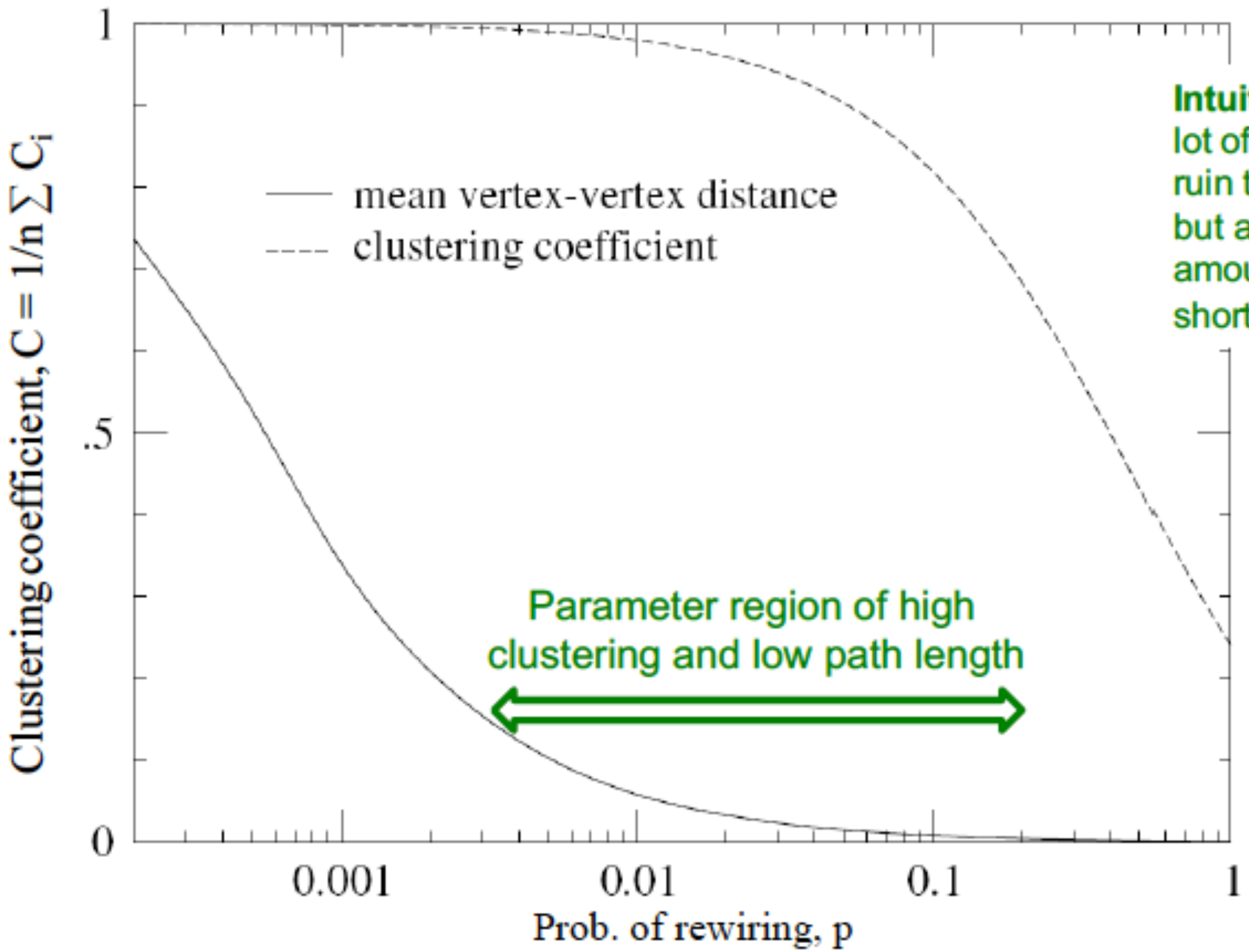
- **Could a network with high clustering be at the same time a small world?**



High clustering High diameter	High clustering Low diameter	Low clustering Low diameter
$h = \frac{N}{2\bar{k}}$ $C = \frac{3}{4}$		$h = \frac{\log N}{\log \alpha}$ $C = \frac{\bar{k}}{N}$

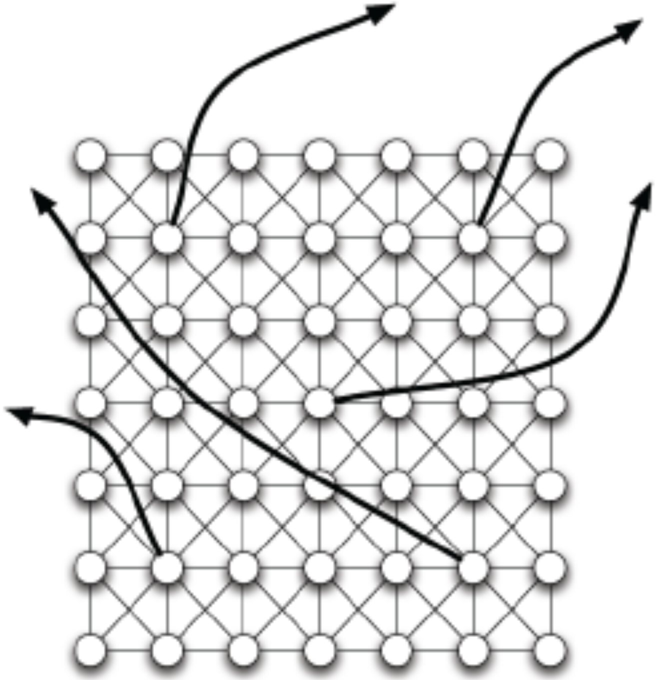


Lattice
1
0.8
0.6
0.4
0.2
0
C(p) / C(0)
L(p) / L(0)
0.0001
0.001
0.01
0.1
1
p
Random Nets



Intuition: It takes a lot of randomness to ruin the clustering, but a very small amount to create shortcuts.

- **Alternative formulation of the model:**
  - Start with a square grid
  - Each node has 1 random long-range edge
    - Each node has 1 spoke. Then randomly connect them.



$$C_i = \frac{2 \cdot e_i}{k_i(k_i - 1)} \geq \frac{2 \cdot 12}{9 \cdot 8} \geq 0.33$$

There are already 12 triangles in the grid and the long-range edge can only close more.

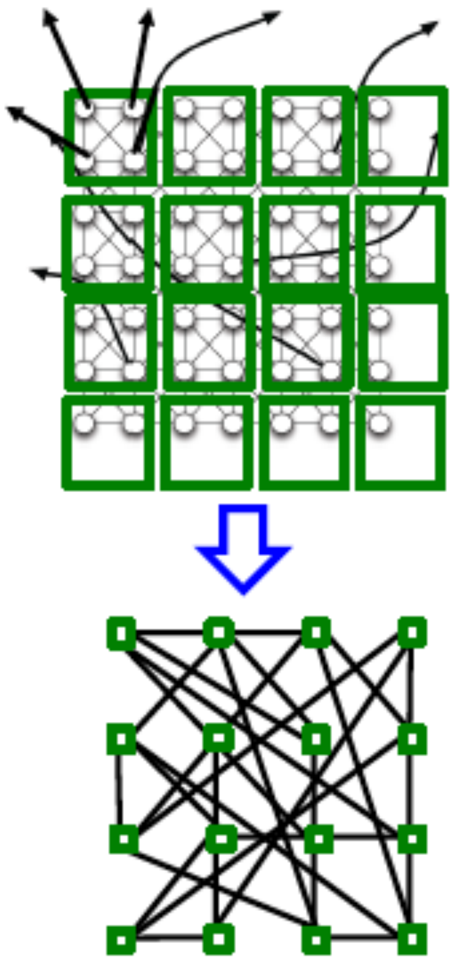
What's the diameter?

It is *O(log(n))*

Why?

- **Proof:**
  - Consider a graph where we contract 2x2 subgraphs into supernodes
  - Now we have 4 random edges sticking out of each supernode
    - **4-regular random graph!**
  - From Thm. we have short paths between super nodes (due to 4 random edges)
  - We can turn this into a path in a real graph by adding at most 2 steps per long range edge (by having to traverse internal nodes)

$\Rightarrow$ **Diameter of the model is** $O(2 \log n)$

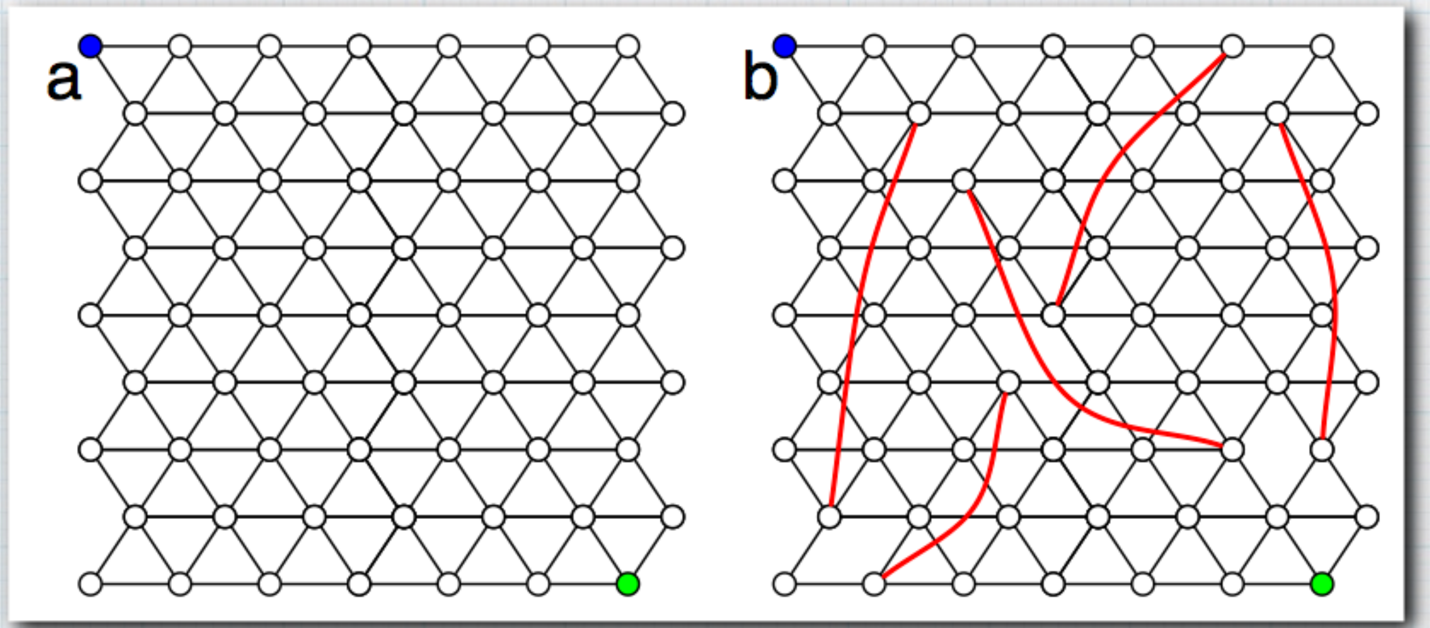


4-regular random graph

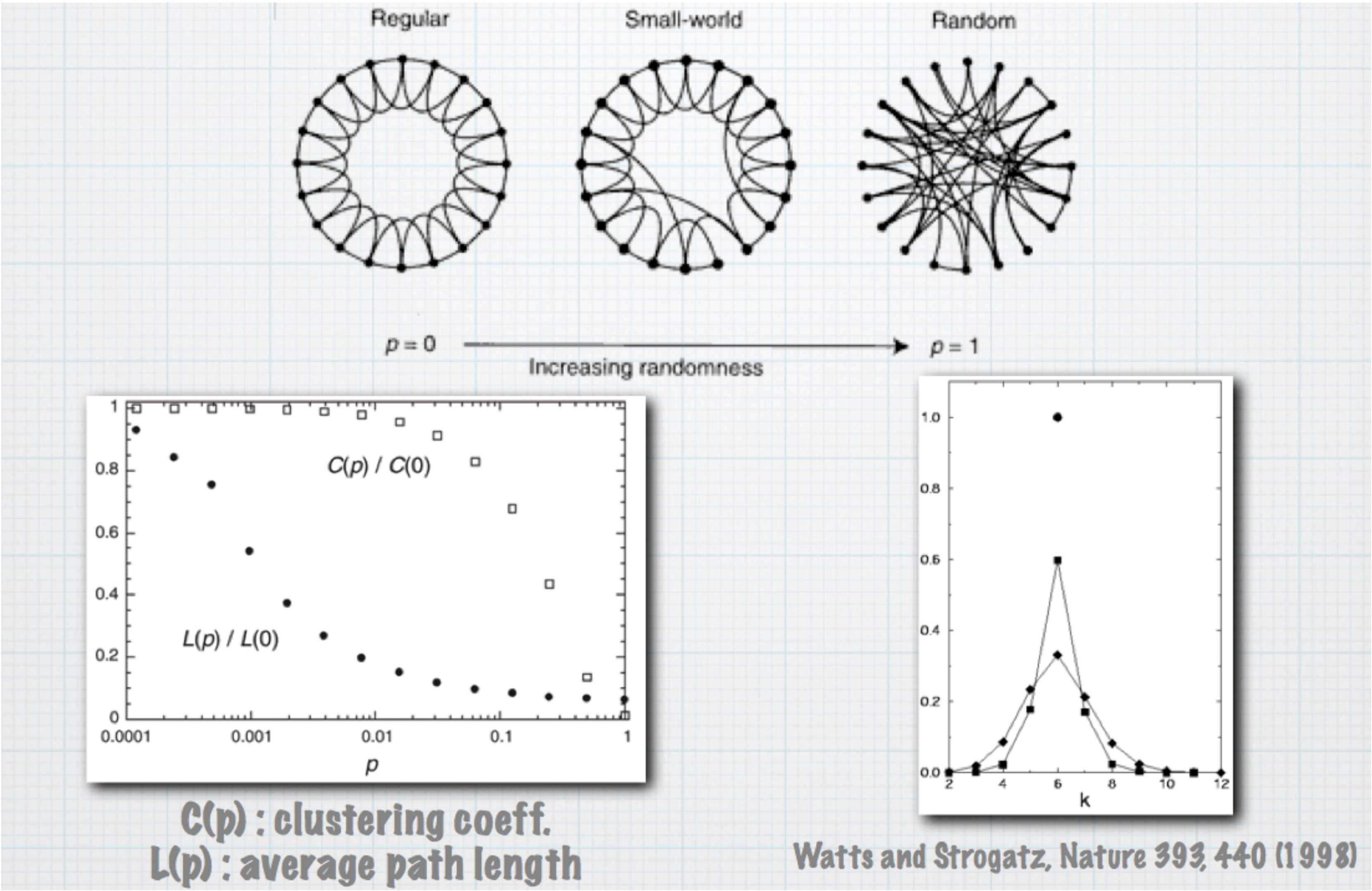
- **Could a network with high clustering be at the same time a small world?**
  - Yes! You don't need more than a few random links
- **The Watts Strogatz Model:**
  - Provides insight on the interplay between clustering and the small-world
  - Captures the structure of many realistic networks
  - Accounts for the high clustering of real networks
  - Does not lead to the correct degree distribution
  - Does not enable **navigation** (next)

Our plan

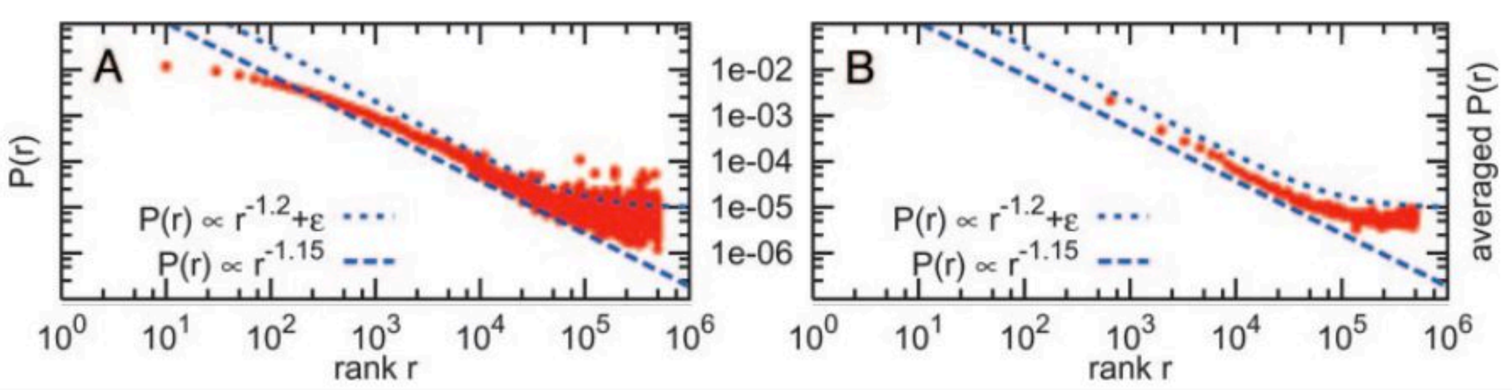
* What happens to the diameter and apl?
* What happens to the clustering coefficient?



Our plan


Regular
Small-world
Random
p = 0
Increasing randomness
p = 1
C(p) / C(0)
L(p) / L(0)
0.0001
0.001
0.01
0.1
1
p
k
C(p) : clustering coeff.
L(p) : average path length
Watts and Strogatz, Nature 393, 440 (1998)

Geographic Linking in the LiveJournal Social Network

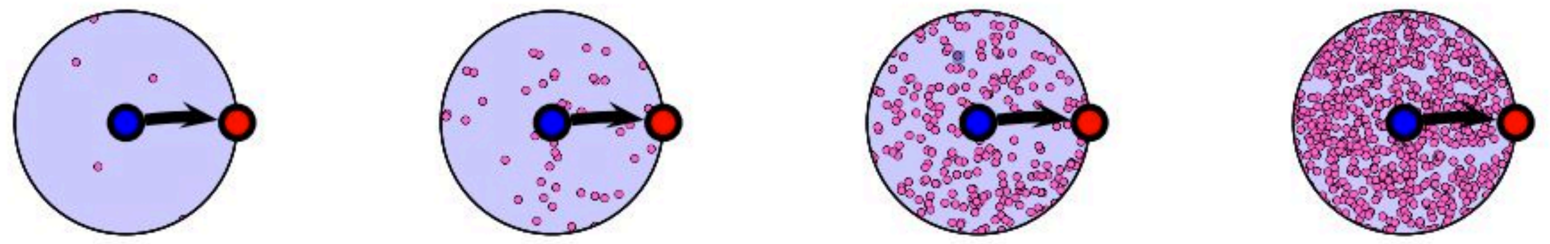


- **Fig. 5A.** The LiveJournal data contain geographic information limited to the level of towns and cities, our data do not have sufficient resolution to distinguish between all pairs of ranks.
- **Fig. 5B.** We show the same data, where the probabilities are averaged over a range of 1,306 ranks.
- This experiment validates that the LiveJournal social network does exhibit rank-based friendship, which thus yields a sufficient explanation for the experimentally observed navigability properties.

http://slideplayer.com/slide/6394623/

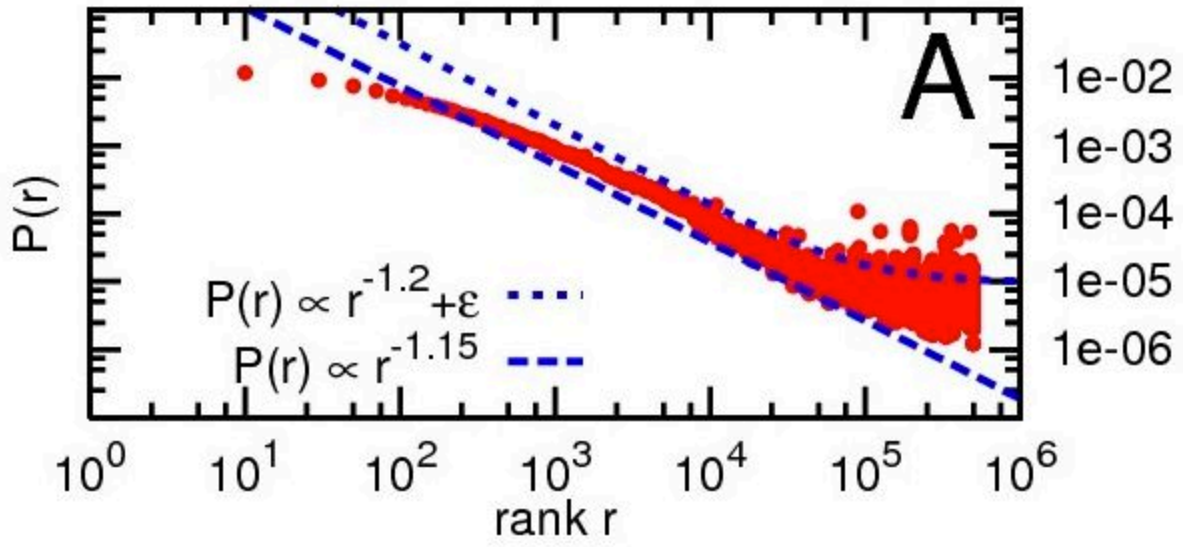
Rank-Based Friendship

- Rank-based friendship implies that GEOGREEDY will find short paths in any social network.
- The LiveJournal network exhibits rank-based friendship.



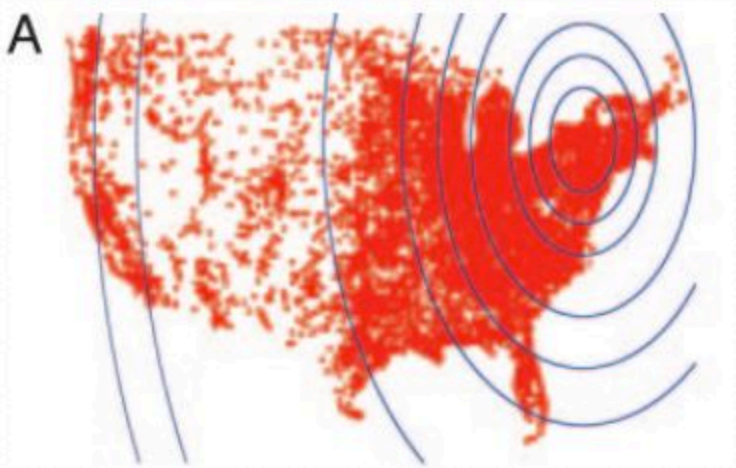
LiveJournal is a searchable network

- Probability that a link exists between two people as a function of the rank between them
  - LiveJournal is a rank-based network → it is searchable



Explain the contradiction

- Evidence of the nonuniformity of the LiveJournal population:



A dot is shown for every distinct United States location home to at least one LiveJournal user. The population of each successive displayed circle increases by 50,000 people. Note that the gap between the 350,000- and 400,000-person circles encompasses almost the entire Western United States.

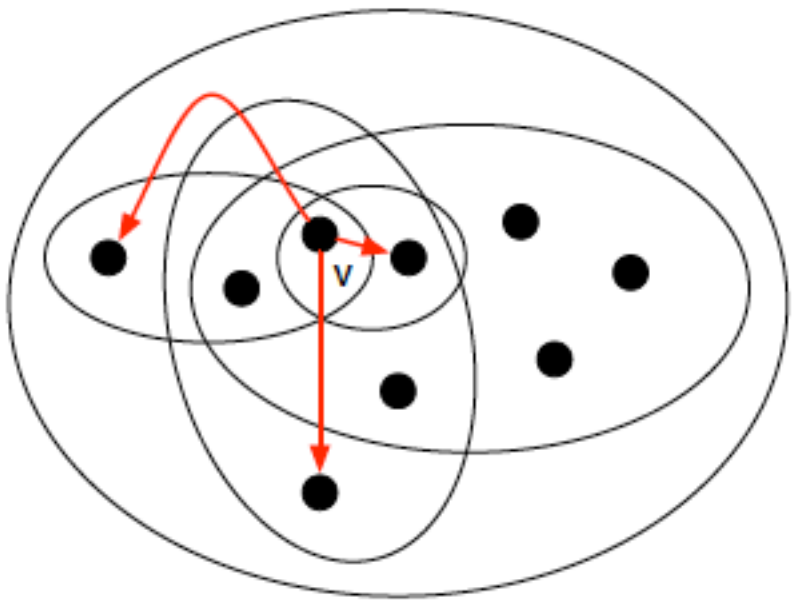


Figure 20.11: When nodes belong to multiple foci, we can define the social distance between two nodes to be the smallest focus that contains both of them. In the figure, the foci are represented by ovals; the node labeled v belongs to five foci of sizes $2, 3, 5, 7$, and 9 (with the largest focus containing all the nodes shown).

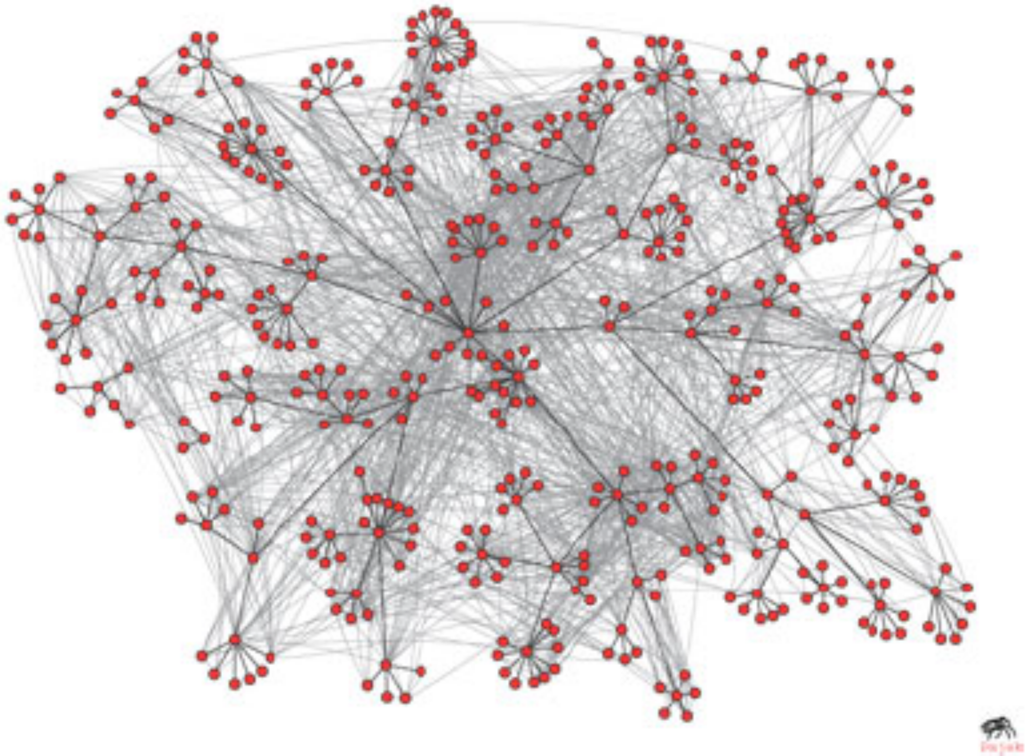


Figure 20.12: The pattern of e-mail communication among 436 employees of Hewlett Packard Research Lab is superimposed on the official organizational hierarchy, showing how network links span different social foci [6]. (Image from http://www-personal.umich.edu/ ladamic/img/hplabsemailhierarchy.jpg)

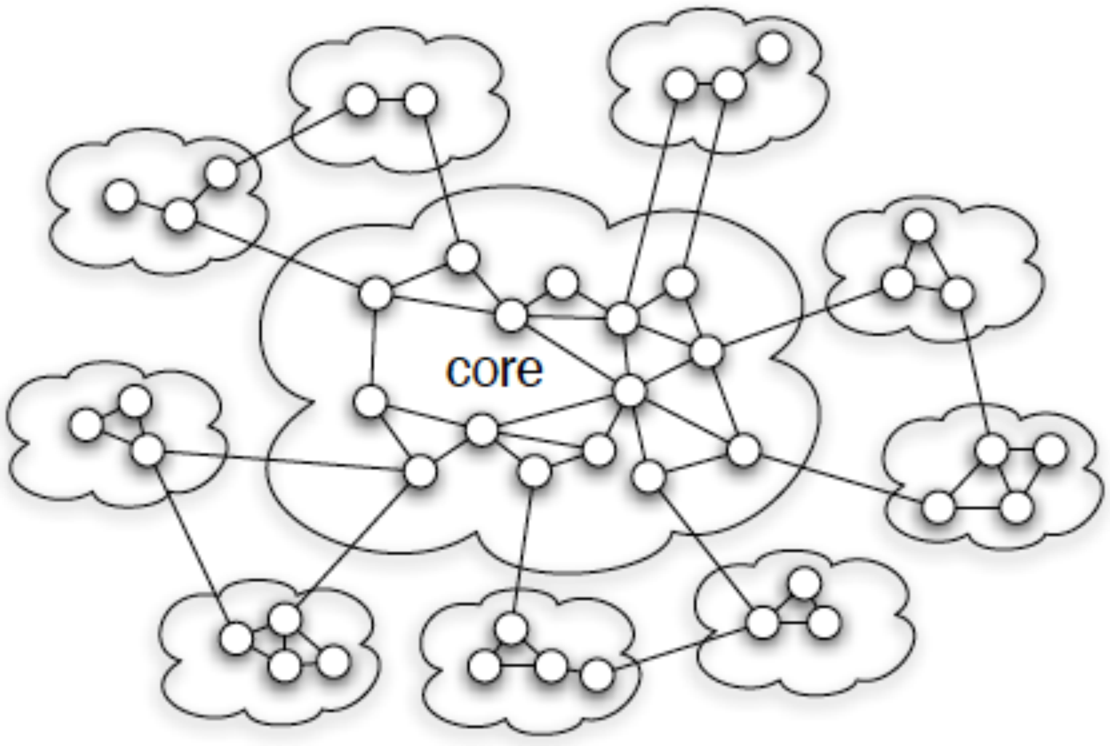


Figure 20.13: The core-periphery structure of social networks.