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importance of feature f_k in o_i

probability that o_j is a document having feature f_k

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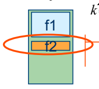

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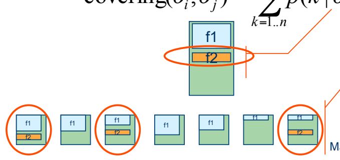

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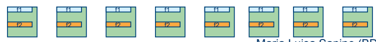
- Suppose the database is a perfect cluster
  - features are uniformly distributed
  - all document are equally likely to be selected



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- Suppose the database is a single cluster

$$\text{covering}(o_i,o_j)=\sum_{k=1..n}\frac{1}{n}\frac{1}{D}=n\frac{1}{n}\frac{1}{D}=\frac{1}{D}$$

How can we estimate the number of clusters in a database?

- Suppose the database is a single cluster

$$\text{covering}(o_i, o_j) = \sum_{k=1..n} \frac{1}{n}\frac{1}{D} = n\frac{1}{n}\frac{1}{D} = \frac{1}{D}$$

- Let's sum up all self-coverings, then

$$\sum_{o_i} \text{covering}(o_i, o_i) = \sum_{D} \frac{1}{D} = 1$$

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How can we estimate the number of clusters in a database?

$$\sum_{o_i} \text{covering}(o_i, o_i) = p$$

There are approximately p clusters

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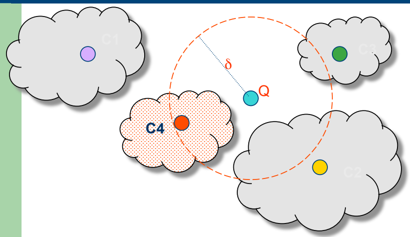
Use of clusters (prune search space)



- …eliminate clusters based on their representatives

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Use of clusters Binary independent features

- Each document is a binary vector
- Documents are organized into clusters
- Each cluster has a representative

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Use of clusters Binary independent features

- Each document is a binary vector
- Documents are organized into clusters
- Each cluster has a representative

- Goal: for each cluster, estimate # of documents having *t* or more matching keywords with a query with *k* keywords

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Use of clusters
Binary independent features

- Each document is a binary vector
- Documents are organized into clusters
- Each cluster has a representative

$$o_i = \langle f_{i,1}, f_{i,2}, \dots f_{i,n} \rangle;$$

Probability that a document in the cluster has this keyword

$$R_O = \langle r_1, r_2, \dots r_n \rangle = \frac{\sum_{o_i \in O} o_i}{|O|}$$

$$q = \langle 1,1,1,\dots,1,0,0,\dots,0 \rangle; with \quad k \quad 1s$$

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$$p(o^k \in O) = \prod_{j=1}^{k} (r_j)^{f_j} (1 - r_j)^{1-f_j}$$

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Use of clusters
Binary independent features

$$o_i = \langle f_{i,1}, f_{i,2}, .. f_{i,n} \rangle; \qquad o^k = \langle f_1, f_2, ..., f_k \rangle;$$

$$R_O = \langle r_1, r_2, .. r_n \rangle = \frac{\sum_{o_i \in O} o_i}{|O|}$$

$$num(t,Q) = \sum_{o^k \text{ with } t \text{ 1s}} \left(p(o^k \in O) \right)$$

Use of clusters
Non-binary, independent features

- Each document is a non-binary vector
- Documents are organized into clusters
- Each cluster has a representative

- Goal: for each cluster, find the probability that one object in the cluster will be more than S similiar to the query

Use of clusters
Non-binary independent features

$$o_i = \langle f_{i,1}, f_{i,2}, .. f_{i,n} \rangle; \qquad o^k = \langle f_1, f_2, ..., f_k \rangle;$$

Use of clusters
Non-binary independent features

$o_i = \langle f_{i,1}, f_{i,2}, \ldots f_{i,n} \rangle;$ $\quad o^k = \langle f_1, f_2, \ldots, f_k \rangle;$

$R_O = \langle [r_1, w_1][r_2, w_2] \ldots, [r_n, w_n] \rangle$

$q = \langle q_1, q_2, \ldots, q_k \rangle$

Probability that a document in the cluster has this keyword

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Use of clusters
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$o_i = \langle f_{i,1}, f_{i,2}, \ldots f_{i,n} \rangle;$ $\quad o^k = \langle f_1, f_2, \ldots, f_k \rangle;$

$R_O = \langle [r_1, w_1][r_2, w_2] \ldots, [r_n, w_n] \rangle$

The average weight of the keyword in the documents that have this keyword

$q = \langle q_1, q_2, \ldots, q_k \rangle$

Probability that a document in the cluster has this keyword

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Use of clusters
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$q = \langle q_1, q_2, \ldots, q_k \rangle$

$cont(i, Q) = w_i q_i;$ with r_i probability

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Use of clusters
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$$p(sim(O,Q) = s) = coef\left(x^s, \prod_{i=1}^{k} \left(r_i x^{w_i q_i} + (1 - r_i)\right)\right)$$

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Use of clusters
Non-binary independent features

$o_i = \langle f_{i,1}, f_{i,2}, \ldots f_{i,n} \rangle;$ $o^k = \langle f_1, f_2, \ldots, f_k \rangle;$

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A generating function!..not evaluated

$q = \langle q_1, q_2, \ldots, q_k \rangle$

$$p(sim(O,Q) = s) = coef\left(x^s, \prod_{i=1}^{k} \left(r_i x^{w_i q_i} + (1 - r_i)\right)\right)$$

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What if features are not independent?

- Metric spaces assume that features are independent (orthogonal to each other)
- …what if they are not?

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Latent Semantic Indexing

- Used for hidden (latent) concepts in a given collection
  - mostly for text collections (cosine similarity)!
- Let us have
  - |O| objects
  - Each object o is represented with a vector of size |v| (number of features)

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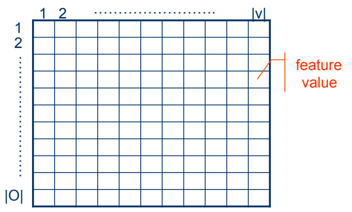
Document-feature vector

OF =



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How can we use this matrix?

- This matrix is the database!!!
- Can we use it to find
  - object-object similarities?
  - feature-feature correlation?
  - independent concepts in the collection?
- Can we use it for efficient indexing?

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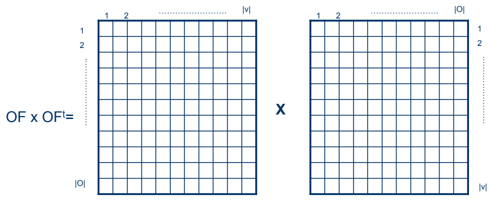
Obj-feature X feature-obj



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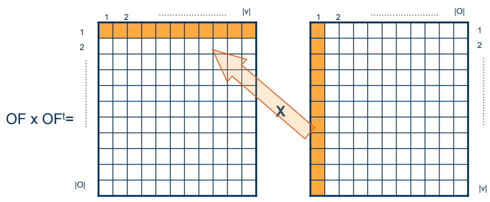

vector multiplication (dot)

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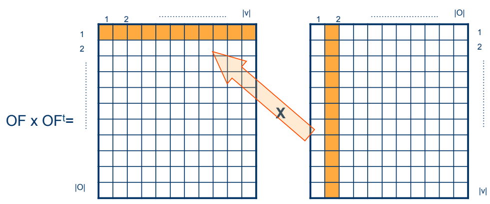
Obj-feature X feature-obj



vector multiplication (dot)

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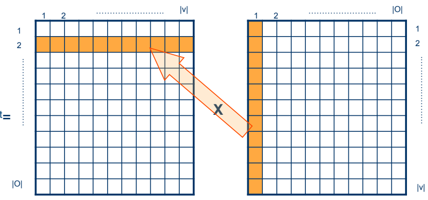
Obj-feature X feature-obj

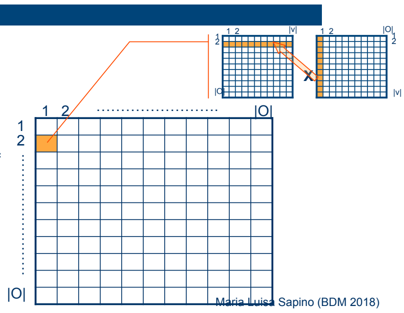
$OF \times OF^t =$



vector multiplication (dot)

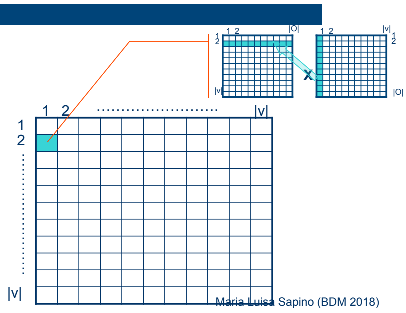
Obj-obj similarity matrix!!!!

$OF \times OF^t = OO =$

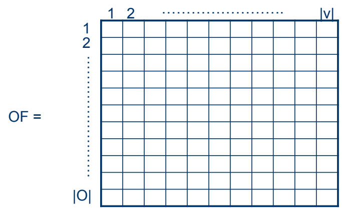


Feature-feature correl. matrix!!!!

$FO \times FO^t = FF =$



Singular valued decomposition



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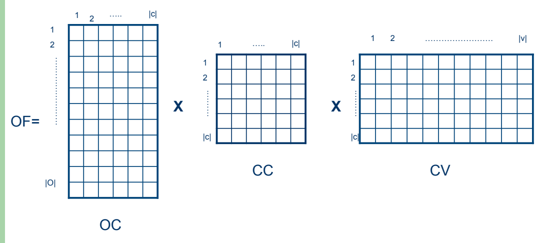

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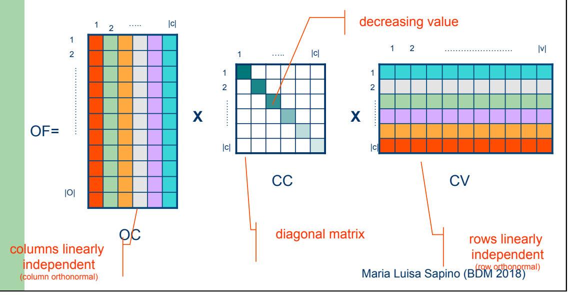
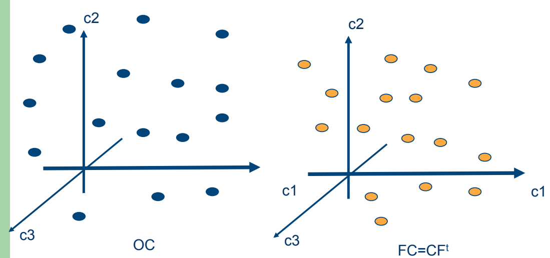


Concept space…and importance



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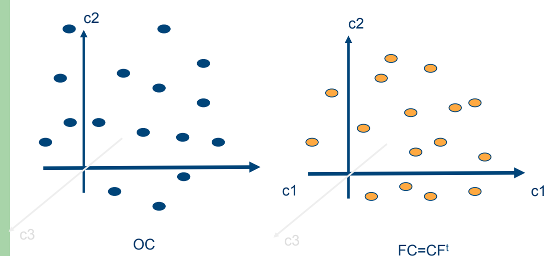
We can ignore less important concepts….



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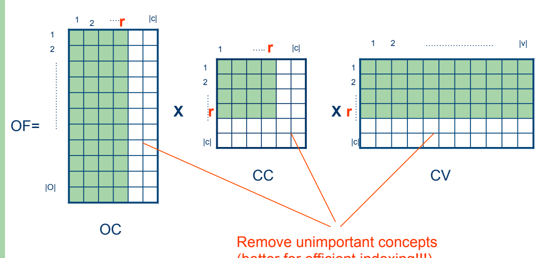
Singular valued decomposition



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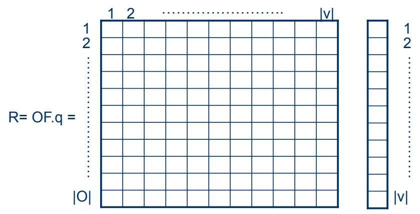
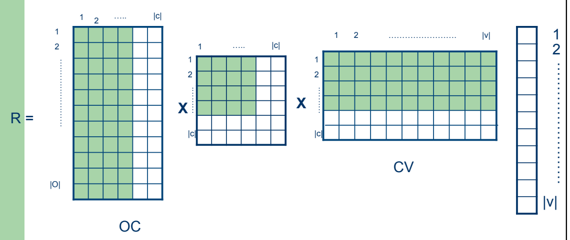
Query processing



Cost = |O|*|v|*|v|

Query processing



Cost = |O|*|r|*|r| + |r|*|r|*|r| + |r|*|v|*|v|