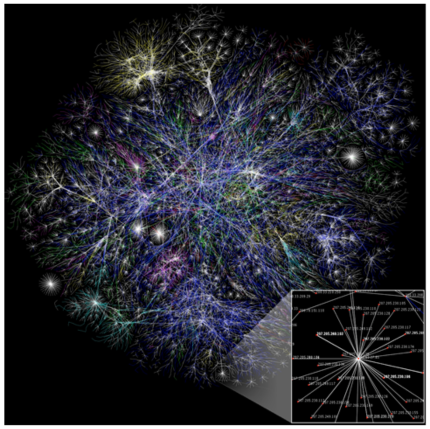
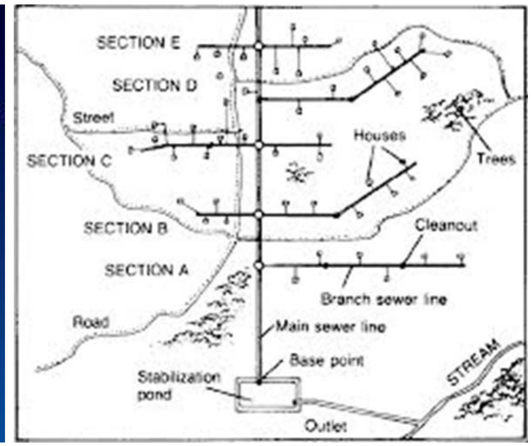
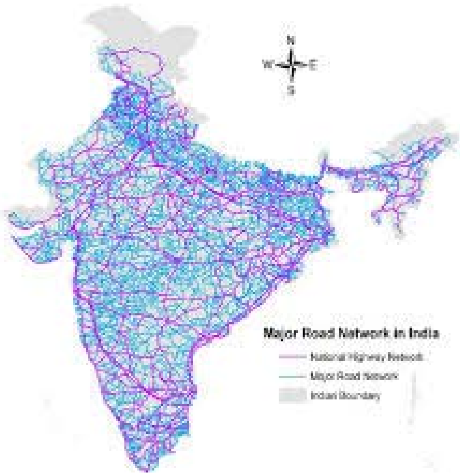
Network as Traffic Carriers

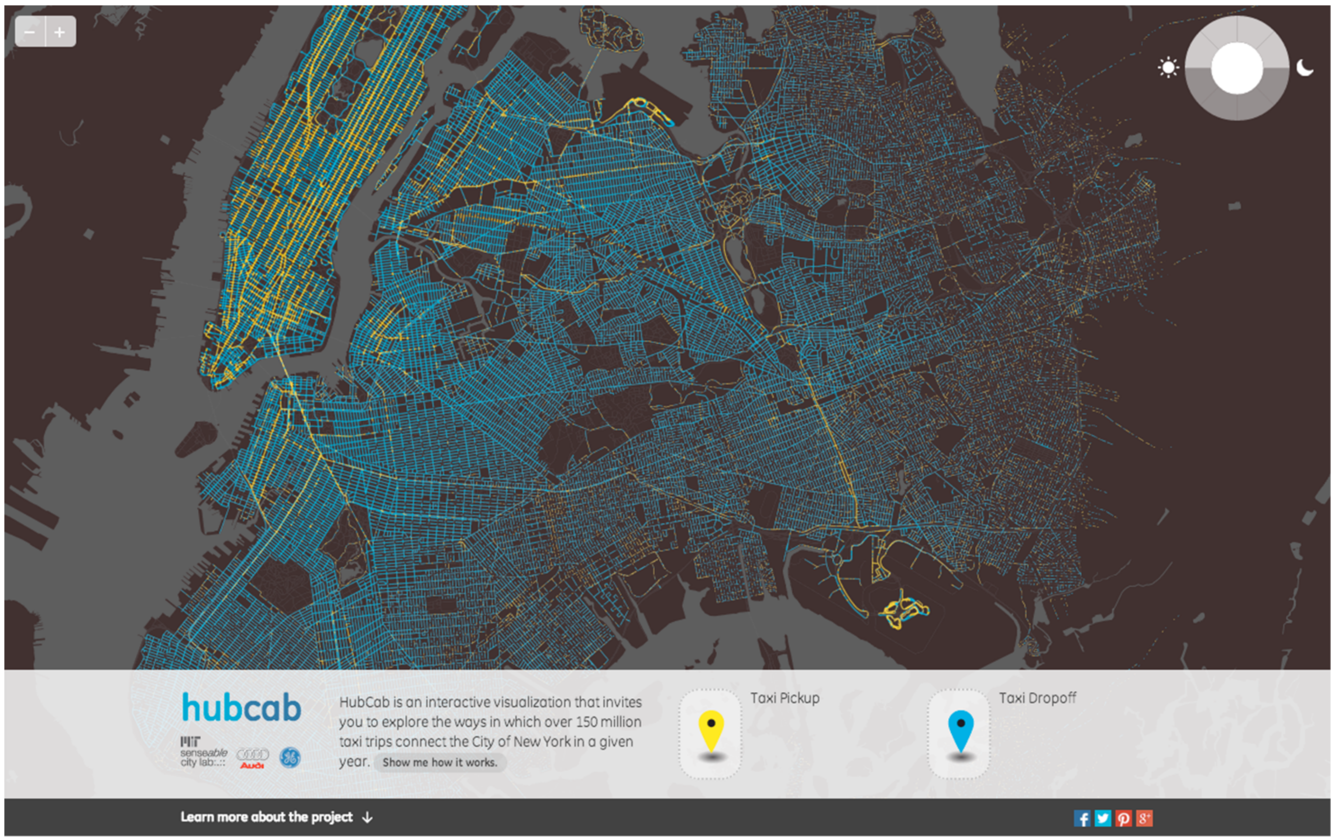




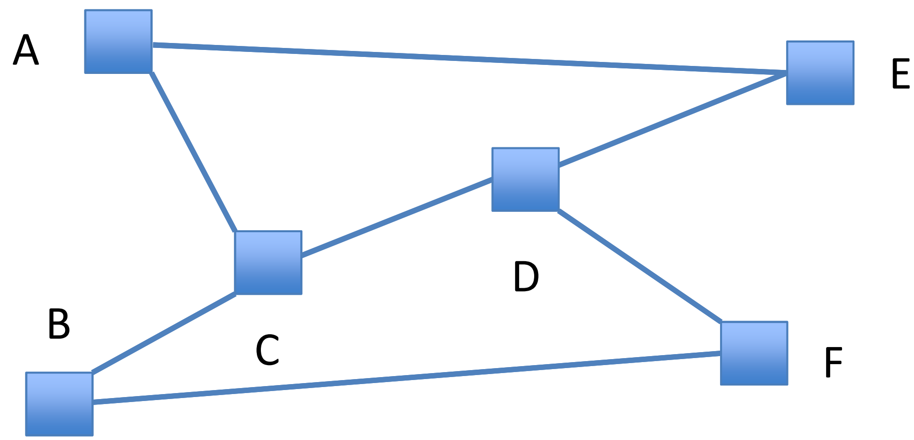


Origin Destination Matrix

See http://www.hubcab.org/



ODM – Computing example

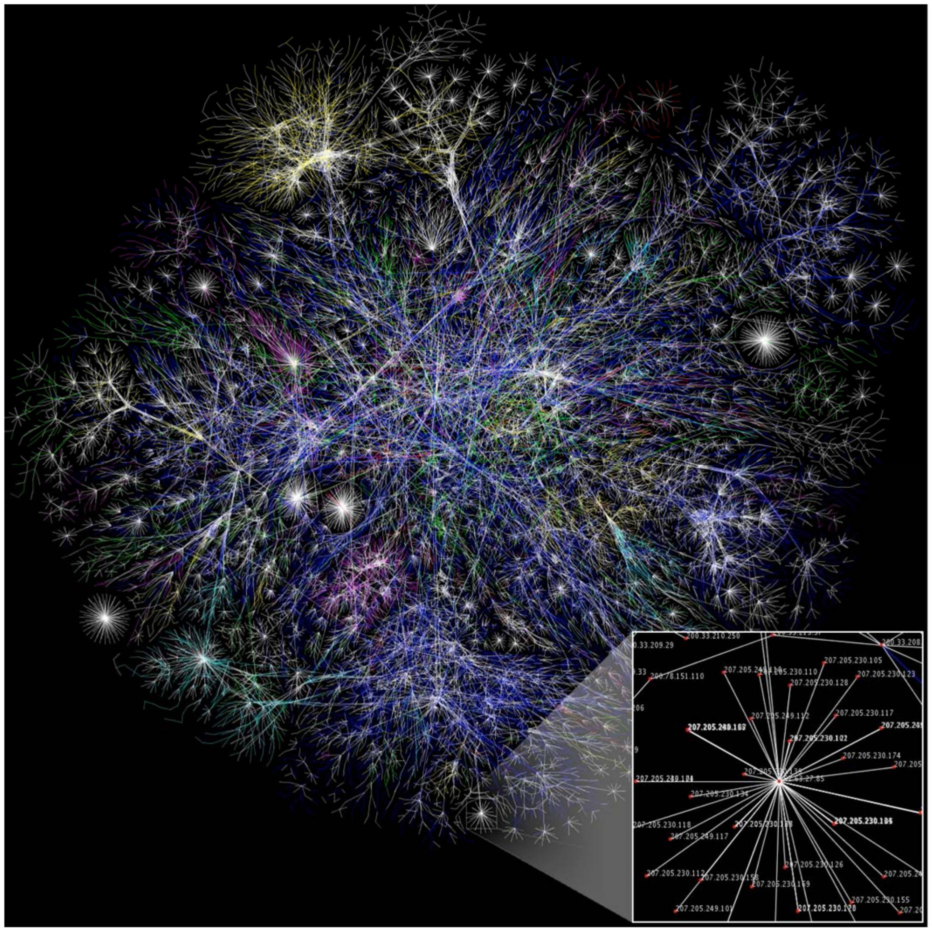


Origins

Destinations

	A	B	C	D	E	F
A	0	3	1	0	2	5
B	2	0	4	1	1	1
C	2	1	0	5	3	2
D				0		
E					0	
F						0

Traffic



Traffic

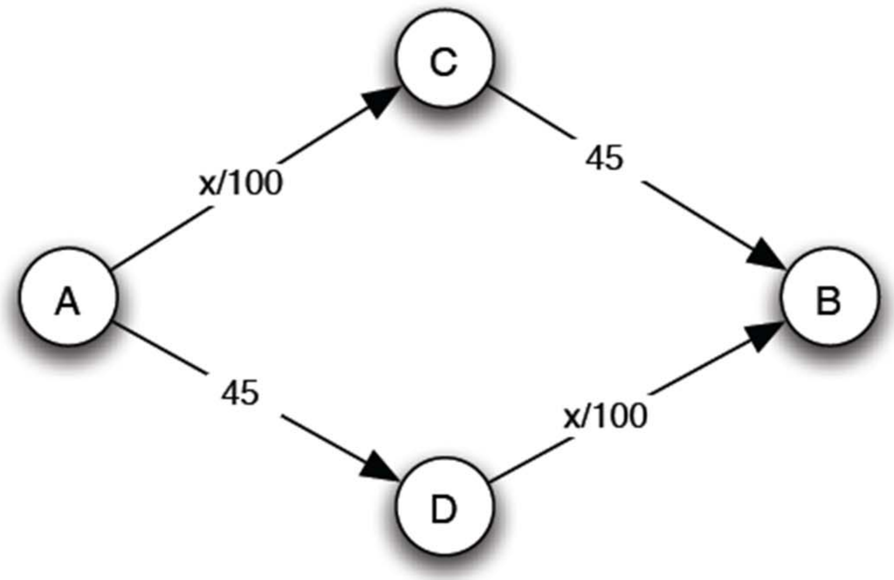


Figure 8.1: A highway network, with each edge labeled by its travel time (in minutes) when there are x cars using it. When 4000 cars need to get from A to B, they divide evenly over the two routes at equilibrium, and the travel time is 65 minutes.

Traffic

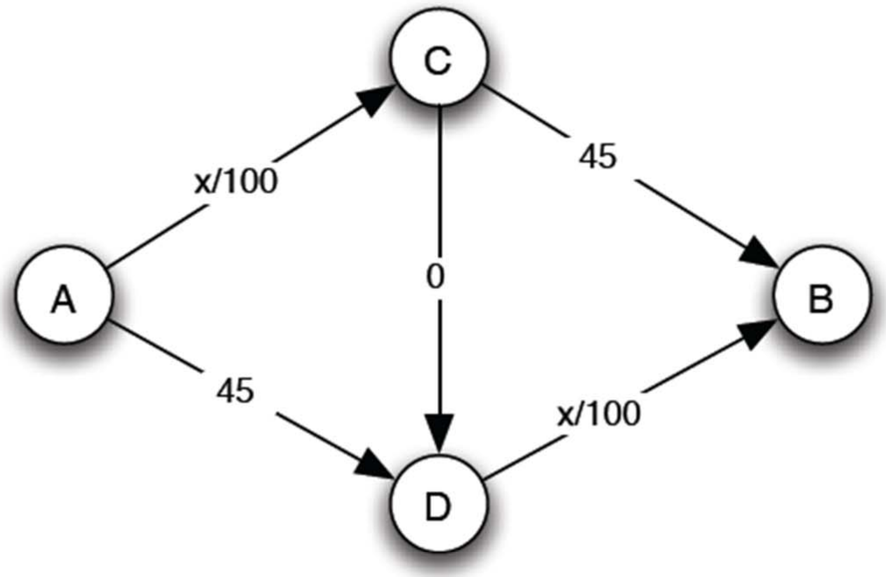
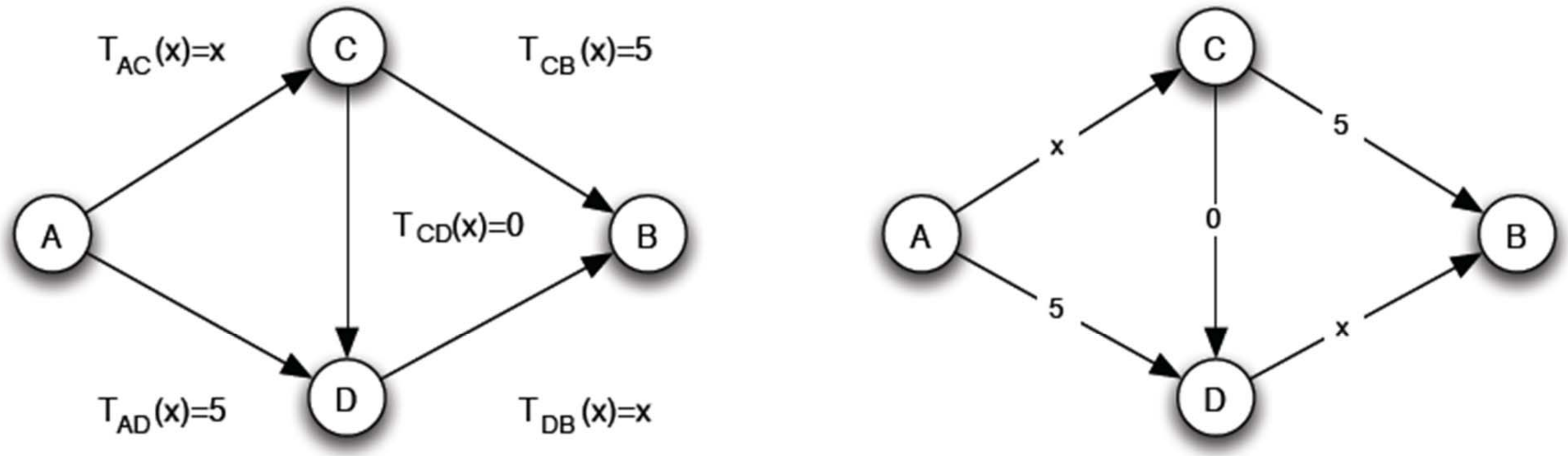


Figure 8.2: The highway network from the previous figure, after a very fast edge has been added from C to D. Although the highway system has been "upgraded," the travel time at equilibrium is now 80 minutes, since all cars use the route through C and D.

Traffic

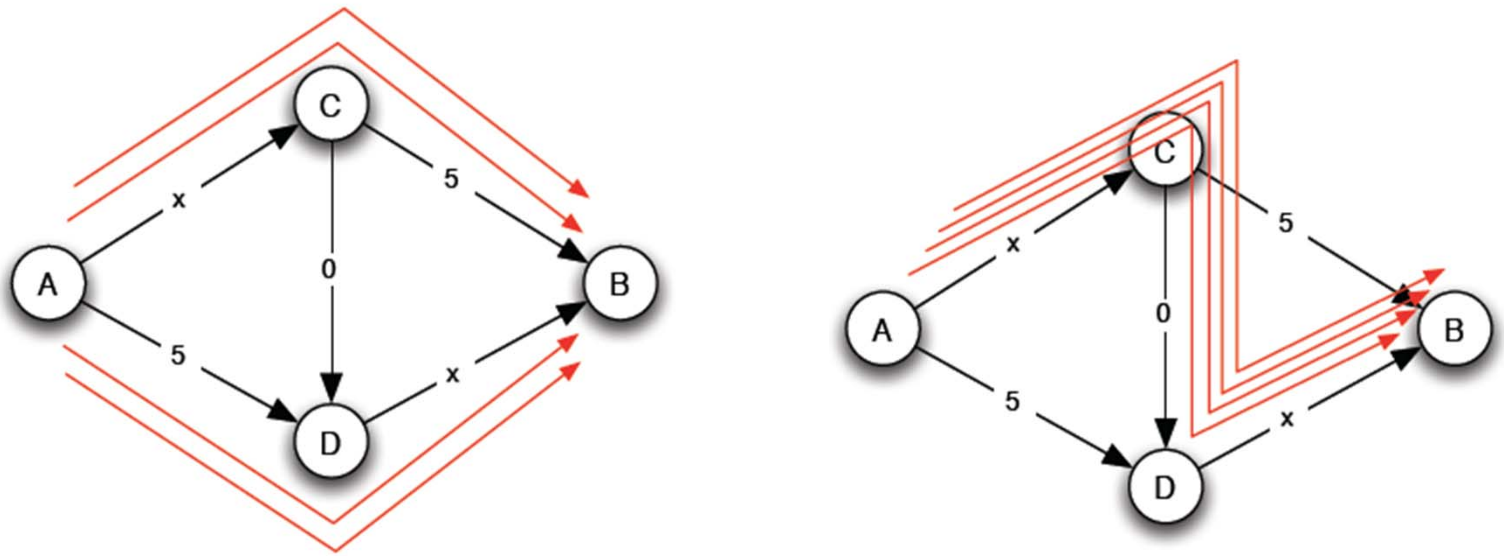


(a) *Travel times written as explicit functions of x.*

(b) *Travel times written as annotations on the edges.*

Figure 8.3: A network annotated with the travel-time function on each edge.

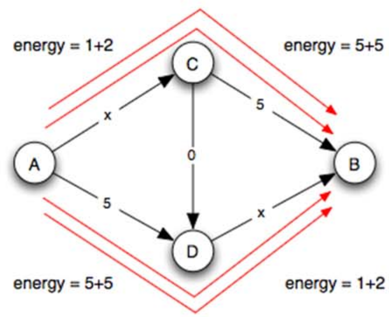
Traffic



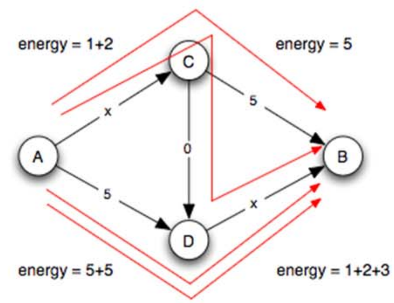
(a) *The social optimum.*　　(b) *The Nash equilibrium.*

Figure 8.4: A version of Braess's Paradox: In the socially optimal traffic pattern (on the left), the social cost is 28, while in the unique Nash equilibrium (on the right), the social cost is 32.

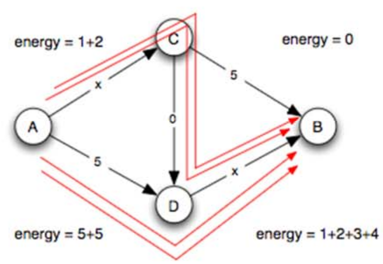
Traffic



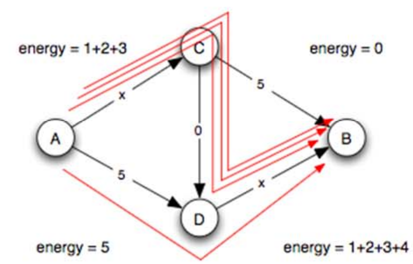
(a) *The initial traffic pattern. (Potential energy is 26.)*



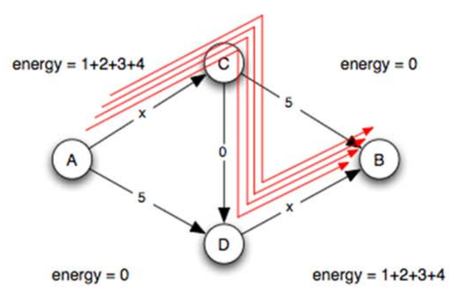
(b) *After one step of best-response dynamics. (Potential energy is 24.)*



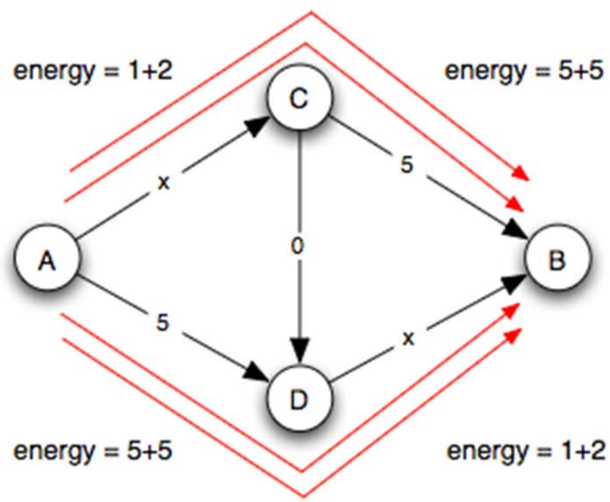
(c) *After two steps. (Potential energy is 23.)*



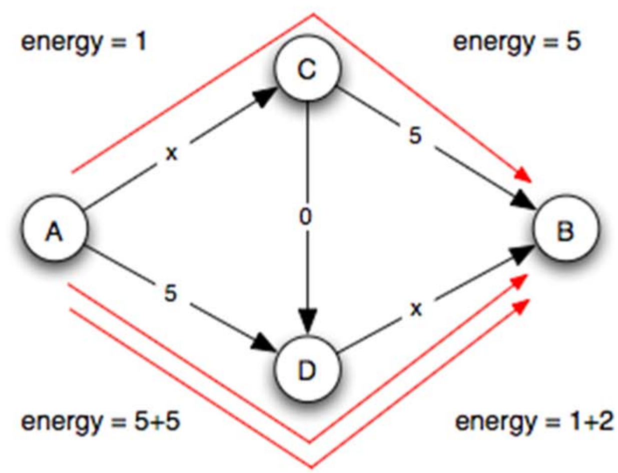
(d) *After three steps. (Potential energy is 21.)*



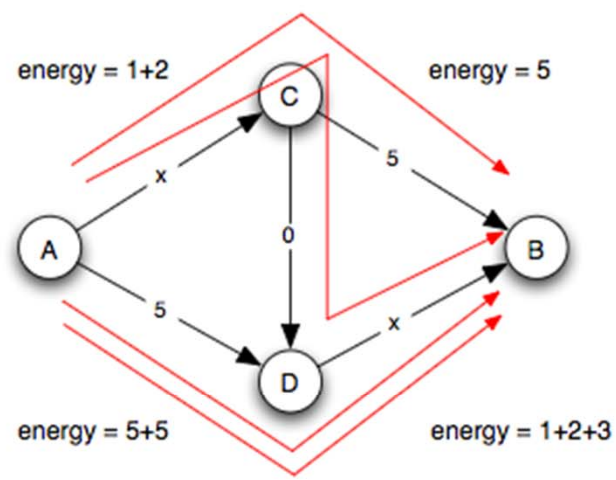
(e) *After four steps: Equilibrium is reached. (Potential*



(a) *The potential energy of a traffic pattern not in equilibrium.*



(b) *Potential energy is released when a driver abandons their current path.*



(c) *Potential energy is put back into the system when the driver chooses a new path.*

Figure 8.6: When a driver abandons one path in favor of another, the change in potential energy is exactly the improvement in the driver's travel time.

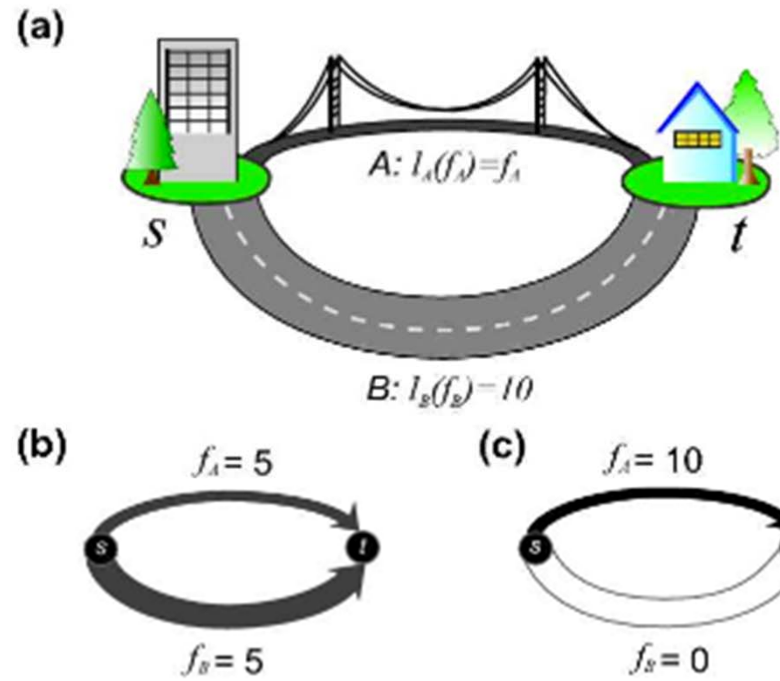


FIG. 1: (color) Illustration of the price of anarchy. (a) Suppose $F = 10$ users travel per unit time from s to t. (b) The socially optimal flow sends five users along each link, thus the total cost is $\mathcal{C} = 75$. (c) In the Nash equilibrium with $f_A = 10$ and $f_B = 0$, $\mathcal{C} = 100$ is higher than in (b).

Traffic

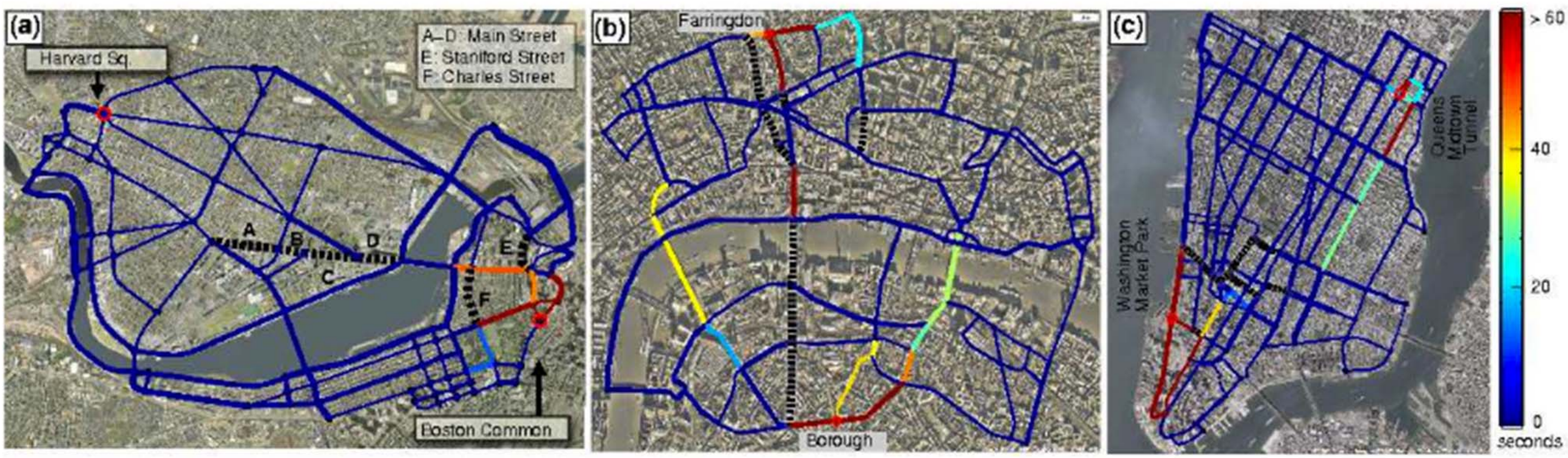


FIG. 2: (color) Networks of principal roads (both solid and dotted lines; the thickness represents the number of lanes). (a) Boston-Cambridge area, (b) London, UK, and (c) New York City. The color of each link indicates the additional travel time needed in the Nash equilibrium if that link is cut (blue: no change, red: more than 60 seconds additional delay). Black dotted lines denote links whose removal reduces the travel time, i.e., allowing drivers to use these streets in fact creates additional congestion. This counter-intuitive phenomenon is called "Braess's paradox."

Traffic

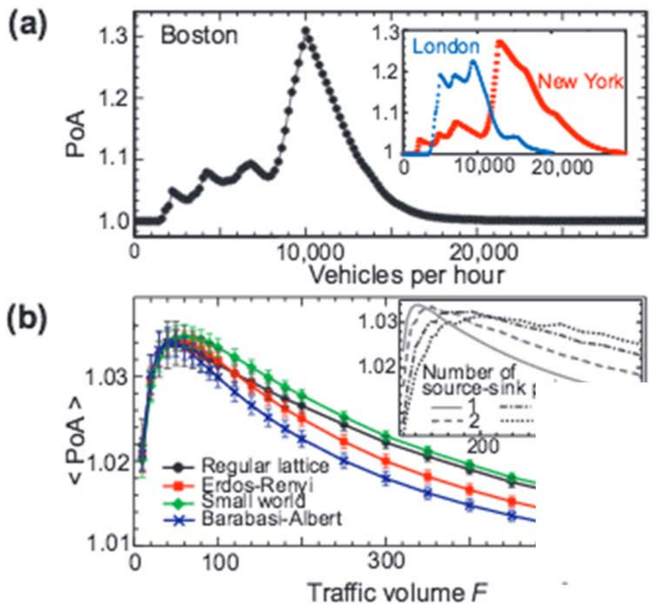
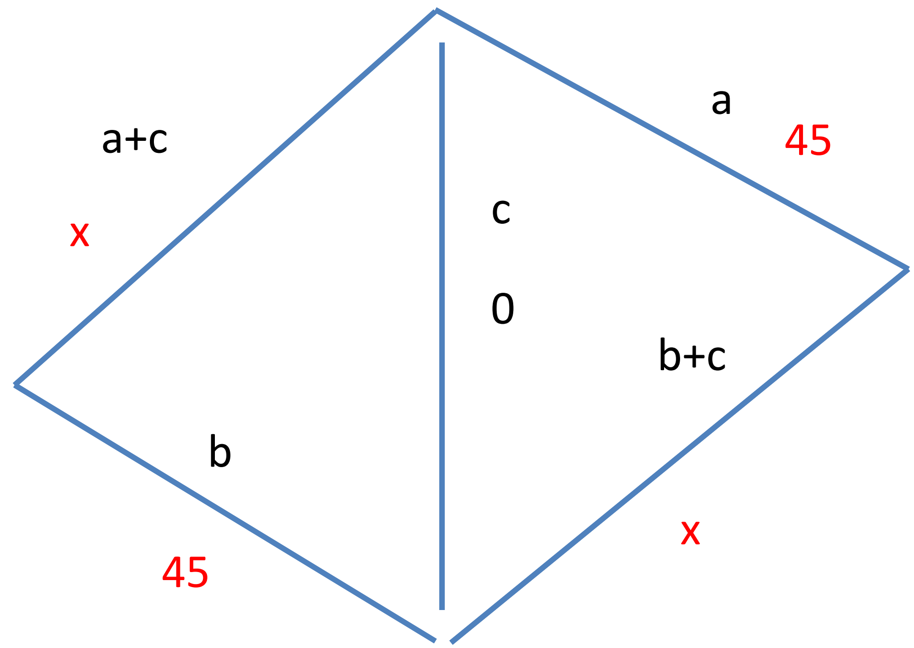


FIG. 3: (color) The price of anarchy (PoA), as a function of the traffic volume F. (a) In Boston's road network for journeys from Harvard Square to Boston Common with BPR delays with $\alpha = 0.2$, $\beta = 10$. Inset: the PoA in London from Borough to Farringdon, and in New York from Washington Market Park to Queens Midtown Tunnel. (b) The PoA in ensembles of model networks with affine delays. All networks have 100 nodes and 300 undirected links. The error bars represent one standard deviation in the PoA-distribution. Inset: the PoA in regular lattices with multiple random sources and sinks ("multi-commodity flows") averaged over 100 to 400 networks. Each pair contributes equally to F.

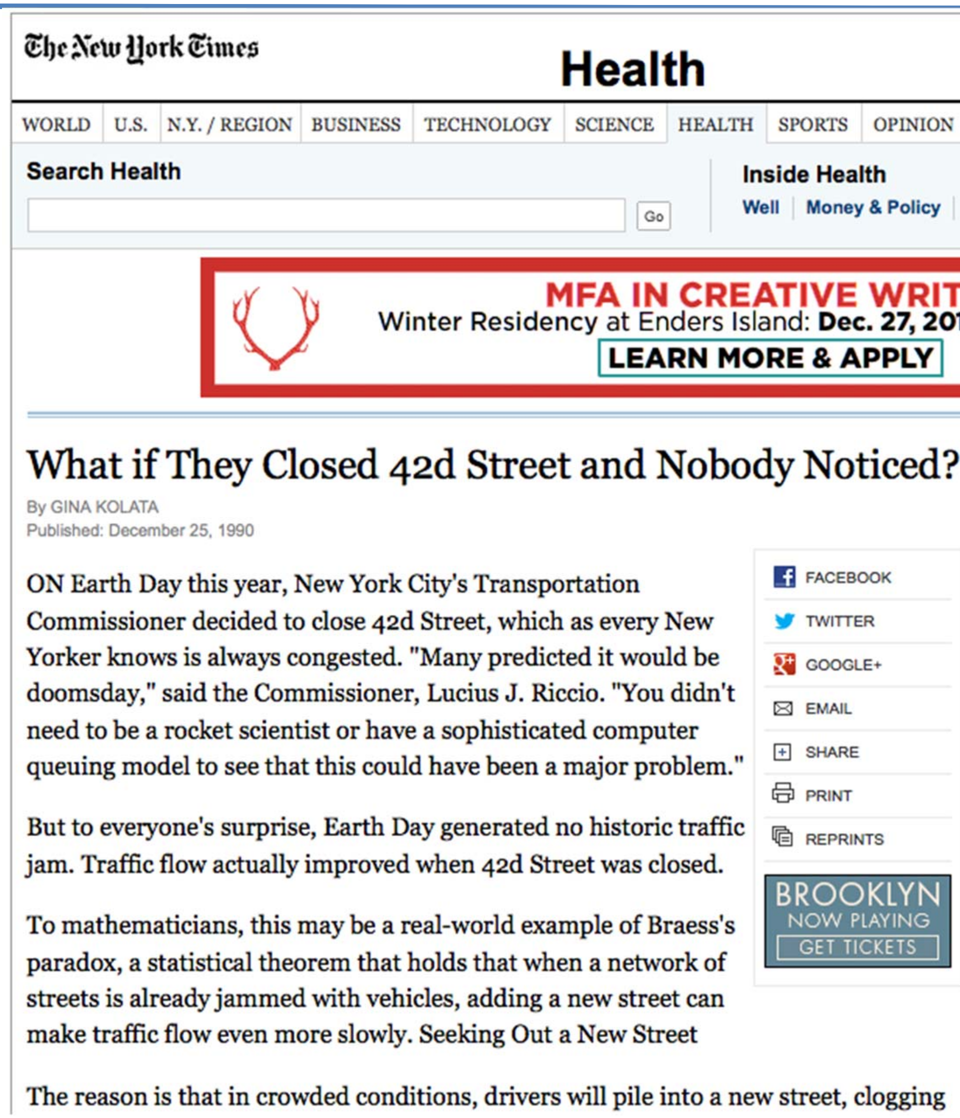
$$\text{PoA} = \frac{\sum l_{ij}(f_{ij}^{NE}) \cdot f_{ij}^{NE}}{\sum l_{ij}(f_{ij}^{SO}) \cdot f_{ij}^{SO}}. \tag{1}$$



$$M < 22.5 \Rightarrow c = M, a = b = 0$$
$$22.5 < M < 45 \Rightarrow c = 45 - M, a = b = (2M - 45)/2$$
$$M > 45 \Rightarrow c = 0, a = b = M/2$$

$$M < 45 \Rightarrow c = M, a = b = 0$$
$$45 < M < 90 \Rightarrow c = 90 - M, a = b = (2M - 90)/2$$
$$M > 90 \Rightarrow c = 0, a = b = M/2$$

Traffic



The New York Times

Health

What if They Closed 42d Street and Nobody Noticed?

By GINA KOLATA
Published: December 25, 1990

ON Earth Day this year, New York City's Transportation Commissioner decided to close 42d Street, which as every New Yorker knows is always congested. "Many predicted it would be doomsday," said the Commissioner, Lucius J. Riccio. "You didn't need to be a rocket scientist or have a sophisticated computer queuing model to see that this could have been a major problem."

But to everyone's surprise, Earth Day generated no historic traffic jam. Traffic flow actually improved when 42d Street was closed.

To mathematicians, this may be a real-world example of Braess's paradox, a statistical theorem that holds that when a network of streets is already jammed with vehicles, adding a new street can make traffic flow even more slowly. Seeking Out a New Street

The reason is that in crowded conditions, drivers will pile into a new street, clogging